Basic terminology and formulas for complex variables

modulus: the length r of z, denoted $|z|$,

argument: angle θ of z (to real axis), denoted $\arg(z)$,

real part of z: x coordinate of z, $\mathrm{Re}(z)$,

imaginary part of z: y coordinate, $\mathrm{Im}(z)$,

complex conjugate of z: reflection of z in the real axis, in other words, negate the imaginary part, denoted $\bar{z}$ or z^*.

Here are some useful things you should convince yourself of, either algebraically or geometrically or both. Drawing little vector pictures helps. For these you can generally assume $r = |z|$, and $\theta = \arg z$.

1. $\mathrm{Re}(z) = \frac{1}{2}[z + \bar{z}]$.
2. $\mathrm{Im}(z) = \frac{1}{2i}[z - \bar{z}]$.
3. $|z| = \sqrt{x^2 + y^2}$.
4. $\tan(\arg z) = \frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}$.
5. $z\bar{z} = |z|^2$.
6. $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$, where $\theta = \arg z$.
7. $\frac{1}{z} = \frac{1}{r}e^{-i\theta}$.
8. $\frac{1}{z} = \frac{1}{x+iy} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$.
9. $\frac{1}{z} = \frac{\bar{z}}{z\bar{z}}$ (rewrite of above).
10. $\frac{z_1}{z_2} = \frac{r_1}{r_2}e^{i(\theta_1 - \theta_2)}$.
11. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$.
12. $\overline{z_1 z_2} = (\bar{z}_1)(\bar{z}_2)$.
13. $\overline{z_1/z_2} = \bar{z}_1/\bar{z}_2$.
14. The *generalized triangle inequality:*

$$|z_1 + z_2 + \cdots + z_n| \leq |z_1| + |z_2| + \cdots + |z_n|.$$