PHYS4340, Test 2, Form: A

Name: ______________________

Date: ______________________

This is due Tuesday, April 17. You are to work on your own on this test.

For the first two following problems, use either Stoke's theorem or the divergence theorem to evaluate each of the integrals in the easiest possible way ($\hat{\mathbf{n}}$ are surface normals):

1. $$\int_S \vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\, dS$$
   over the closed surface of the tin can bounded by $x^2 + y^2 = 9$, $z = 0$, $z = 5$, if
   $$\vec{\mathbf{V}} = 2xy\hat{\mathbf{x}} - y^2\hat{\mathbf{y}} + (z + xy)\hat{\mathbf{z}}.$$

2. $$\int_S (\nabla \times \vec{\mathbf{V}}) \cdot \hat{\mathbf{n}}\, dS$$
   over any surface whose bounding curve is in the (x, y) plane, where
   $$\vec{\mathbf{V}} = (x - x^2 z)\hat{\mathbf{x}} + (yz^3 - y^2)\hat{\mathbf{y}} + (x^2 y - xz)\hat{\mathbf{z}}.$$

3. In your text (Snieder) Read section 10.6, do problems 10.6 a,b,c.

4. Use the Cauchy-Riemann equations to determine which of these functions are analytic. If they have singularities, note their locations and check if the function is analytic away from the singularities:
   - A) $\bar{z}$ (the complex conjugate function.)
   - B) $|z|$
   - C) $\cosh z$
   - D) $f(z = x + iy) = \frac{y - ix}{x^2 + y^2}$

5. Use residues to evaluate
   $$\int_0^\infty \frac{\cos 2x\, dx}{9x^2 + 4}$$

6. Use residues to evaluate
   $$\int_{-\infty}^\infty \frac{dx}{x^2 + 4x + 5}$$

7. Use residues to evaluate

$$\int_{-\infty}^{\infty} \frac{\sin x \; dx}{x^2 + 4x + 5}$$