Final Exam, Math Methods, Spring 2007

The following problems are due Wednesday, May 2. They are roughly in the order the topics appear in the text. You must work on your own and include a statement to that effect.

I will be available via email and at our class times all week for questions.

1. A star undergoes oscillations. What is the frequency ω proportional to?

   A very simple set of physical parameters that could be appropriate are: the radius R, the density ρ, and the Newtonian gravitational constant G. Assume that the density is a constant with radius, so the mass does not need to be included independently. Use the Buckingham pi theorem to find a dimensionless expression for the frequency.

2. A) Find the Taylor series of e^{x^2}.

   B) Compute the Taylor series of e^x centered around $x = 2$.

3. Calculate
$$\oint_S (2x\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} + 5\hat{\mathbf{z}}) \cdot \hat{\mathbf{n}}\, dS$$
   over the surface of a sphere of radius 2, center at the origin.

4. Calculate
$$\oint_C (y\hat{\mathbf{x}} - x\hat{\mathbf{y}} + z\hat{\mathbf{z}}) \cdot d\vec{\mathbf{r}}$$
   around the circumference of the circle of radius 2, center at the origin, in the xy plane.

5. Interpret two complex numbers $v_1 = x_1 + iy_1$ and $v_2 = x_2 + iy_2$ as vectors in the xy-plane. Show that
$$(\bar{v}_1)v_2 = (\vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_2) + i(\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2)_{\hat{z}}.$$
   $\bar{v}$ is the complex conjugate $\bar{v} = x - iy$.

   The term with $(\ldots)_{\hat{z}}$ above is the value of the $\hat{z}$-component of the cross product (the only component in this case.)

   Hint: don't overthink!

6. Evaluate both of the following integrals:
$$\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx, \qquad \int_{-\infty}^{\infty} \frac{x \cos \pi x}{x^2 + 2x + 5} dx$$
   using one complex contour integral.

7. Using the forward transform (eq. 15.28 in your text)

$$F(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx$$

find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|/a, & (|x| \le a) \\ 0, & (|x| > a). \end{cases}$$