Curl

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$$\nabla \times \vec{\mathbf{v}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial_x & \partial_y & \partial_z \\ v_x & v_y & v_z \end{vmatrix} = \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix}$$

Notation note: $\partial_y x = \partial y / \partial x$.

Consider a line integral around a tiny surface element. (fig. 7.1) The z component of a curl will be

$$\oint_{dx\,dy} \vec{\mathbf{v}} \cdot d\vec{\mathbf{r}} = \partial_x v_y - \partial_y v_x$$

for v_x and v_y evaluated at x, y.

The component of curl $\vec{\mathbf{v}}$ in a certain direction is the closed line integral of $\vec{\mathbf{v}}$ along a closed path perpendicular to this direction, normalized to unit surface area.

Curl can also be described as *circulation density.*

If $\vec{\mathbf{v}}$ is the velocity of fluid flow field,

$$\omega = \nabla \times \vec{\mathbf{v}}$$

is called the **vorticity.**

Vorticity can come from rotation of the fluid and/or shear.