Total time derivative
Continuous systems
Homework for Ch. 4 and 5

Math Methods, D. Craig

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Total and partial time derivatives

If you're a **stationary observer** monitoring the temperature of the air as the wind goes by, and you detect an increase

$$\frac{\partial T}{\partial t} > 0,$$

this is written with a partial because $T = T(\vec{\mathbf{r}}, t)$ In the definition of the partial derivative time is varied but other variables are fixed.

If you're in a ballon moving through **stationary** T **field** then $\partial T/\partial t = 0$ while you can have

$$\frac{dT}{dt} > 0.$$

This is because you are moving along at $d\vec{\mathbf{r}}/dt = \vec{\mathbf{v}}$.

For the particular case of moving through a stationary $T(\vec{\mathbf{r}})$:

$$\frac{dT(\vec{\mathbf{r}})}{dt} = (\vec{\mathbf{v}} \cdot \nabla T)$$

Problems 5.5 c–f work out the steps for getting the total time deriviative explicitly. In the general case of $f = f(\vec{\mathbf{r}}, t)$, then

$$\frac{df(\vec{\mathbf{r}}, t)}{dt} = \frac{\partial f(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{v}} \cdot \nabla f(\vec{\mathbf{r}}, t)$$

is the **total time derivative.**

It is related to the movement of a quantity. In a gas it may be the movement of a parcel of material carried around by the flow.

In general there are two ways to describe continuous systems (such as fluids):

Eulerian description: describe every quantity at a fixed location in space. Then the change with properties in time is described by $\partial d/\partial t$.

Lagrangian description: follow a property as carried by the flow. Then change is described by total time derivative d/dt.

Eulerian is often most convenient for numerical simulation, however if one must track particles or parcels, Lagrangian description is most useful.

Gradient in spherical coordinates

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

In cylindrical coordinates

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial f}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial f}{\partial z}$$

Homework assignment 3 for Ch. 4 and 5

4.4c Taking the Jacobian with the determinant.

4.5 b and c Cylindrical coordinates. The geometric reasoning is easier than the spherical case.

5.4 f Extending force analysis to power.

5.5 g Showing the balloon and stationary observers both work out with total time derivative.

Due Tuesday Feb. 13 by 3 pm.