Insight from a series: multilayer reflection and Homework assignment 2

Math Methods, D. Craig

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We have one stack of layers on the left with reflection & transmission coefficients R_L, T_L and a stack on the right with R_R, T_R See fig 3.4

What will be T, R for the combined stacks? You might think

$$T \stackrel{?}{=} T_L T_R.$$

But this would be wrong because there are right and left-going waves of relative strength A and B between the stacks that lead to **reverberation terms.**

3.4 Problem a

Working out the relation between all these (see figure and think about reflections):

$$
\begin{aligned}
A &= T_L + BR_L \\
B &= AR_R \\
T &= AT_R \\
R &= R_L + BT_L
\end{aligned}
$$

The first two can be solved for A, B, then substituted into the last two to solve the system.

3.4 problem b

$$
\begin{aligned}
A &= T_L + BR_L \\
A &= T_L + AR_RR_L \\
A(1 - R_RR_L) &= T_L \\
A &= \frac{T_L}{(1 - R_LR_R)}
\end{aligned}
$$

then:

$$
\begin{aligned}
B &= AR_R \text{ so} \\
B &= \frac{T_LT_R}{(1 - R_LR_R)}
\end{aligned}
$$

Why does right-going A have something other than T_L?

$$A = \frac{T_L}{(1 - R_L R_R)}$$

3.4 problem c

Expand:

$$\frac{1}{(1 - R_L R_R)} = 1 + R_L R_R + R_L^2 R_R^2 + \cdots$$

So A is

$$A = T_L + T_L R_L R_R + T_L R_L^2 R_R^2 + \cdots$$

and B is

$$B = T_L R_R + T_L R_L R_R^2 + T_L R_L^2 R_R^3 + \cdots$$

Notice that the terms in the series show the successive reflections. Including $B = A R_R$ gives the terms for the extra reflections from the right for the left-going B wave.

3.4 Problem d Easily substitute in expressions for A, B to get

$$R = R_L + \frac{T_L^2 R_R}{(1 - R_L R_R)}$$
$$T = \frac{T_L T_R}{(1 - R_L R_R)}$$

Problem e

Now we can see that our original supposition is approximately true

$$T \stackrel{?}{=} T_L T_R.$$

if $R_L R_R \to 0$.

Building up stacks by recursion.

Suppose we have a stack of n layers on the left with $R_L = R_n$ and $T_L = T_n$. We add single layer on the right of known $R_R = r_{n+1}$ and $T_R = t_{n+1}$. The new coefficients for the new stack will be

$$R_{n+1} = R_n + \frac{T_n^2 r_{n+1}}{(1 - R_n R_{n+1})}$$

$$T_{n+1} = \frac{T_n t_{n+1}}{(1 - R_n R_{n+1})}$$

This is a wonderful job for a computer, you can build up total coefficients for large stacks of layers this way.

Problem f is just to realize what T_0, R_0 are for "nothing" and then see how to build up from there.

Homework

Work out the Taylor series for two dimensions: **Problems 3.1 f–g.**

Do the complete bouncing ball analysis: **Problems 3.3 a–f.**

Due 5 pm Monday Feb. 5.