Homework 1: Section 2.6 of Snieder

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Your first assignment is to do section 2.6 of Snieder and turn it in by Monday, Jan 29. Here is a slightly abbreviated version for those that lack a text at this point:

2.6

. . . Use dimensional analysis to study the dependence of flow of a viscous fluid through a cylindrical pipe as shown in figure 2.3. The flow is driven by a pressure gradient $\partial p/\partial x$ along the center axis of the cylinder. We assume the fluid has a viscosity μ, and we want to find the relation between the strength of the flow along the pipe per unit time and the radius R. As a measure of the flow rate we use the volume of the flow per unit time, and designate this quantity with the symbol Φ.

Problem a The physical quantities that are of relevance to this problem are the pressure gradient $\partial p/\partial x$, the viscosity μ, the radius R, and the flow rate Φ. Write down the physical dimensions of each of these properties. In order to find the dimension of viscosity you can use the relation $\tau = \mu \partial v/\partial z$, where τ is the shear stress (with the dimensions of pressure), v velocity, and z distance.

Problem b Use the Buckingham pi theorem to show that the flow rate is given by

$$\Phi = constant \frac{\partial p/\partial x}{\mu} R^4. \text{ (2.32 in your text)}$$

Problem c This expression states that the flow rate is proportional to the pressure gradient, which reflects the fact that a stronger pressure gradient generates a stronger driving force for the flow, and hence a stronger flow. Give a similar physical explanation for the dependence of the flow rate on the viscosity and the radius. At first you might think that the flow rate is proportional to the (cross-sectional) surface area πR^2 of the pipe. Try to give a physical explanation of the R^4-dependence of the flow rate on the radius.

This relation can be derived from the Navier-Stokes equations for fluid flow, but we just need a more descriptive explanation. You might check your introductory physics text—some cover this in chapters on fluid flow.