A Child's Guide to Interferometry

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To understand a correlation interferometer it is best to consider a simple one-dimensional case: that of a two-element multiplying interferometer. The following treatment is partly based on one by P. Fisher, similar examples can be found in any radio astronomy text.

Two antennas are separated by an east-west baseline of length B. An astronomical point source is in the eastern sky at an angular elevation of θ above the horizon (see figure 1)

Since the source is practically at an infinite distance, the paths (and wavefronts) from the source to the two antennas are parallel. As figure 1 shows, after passing **antenna 1** the wavefront must pass through an additional distance d to reach **antenna 2**. The signal at **antenna 1** will produce a voltage which can be represented as $E \cos 2\pi f t$ where E is the amplitude of the electric field, f the frequency, and t the time. The same signal will travel to **antenna 2** through the extra distance d, so the wavefronts will reach **antenna 2** at a time d/c later than at **antenna 1**. The voltage at **antenna 2** can be written as:

$$E_2 = E \cos[2\pi f(t - d/c)] \tag{1}$$

or

$$E_2 = E \cos(2\pi f t - \phi) \tag{2}$$

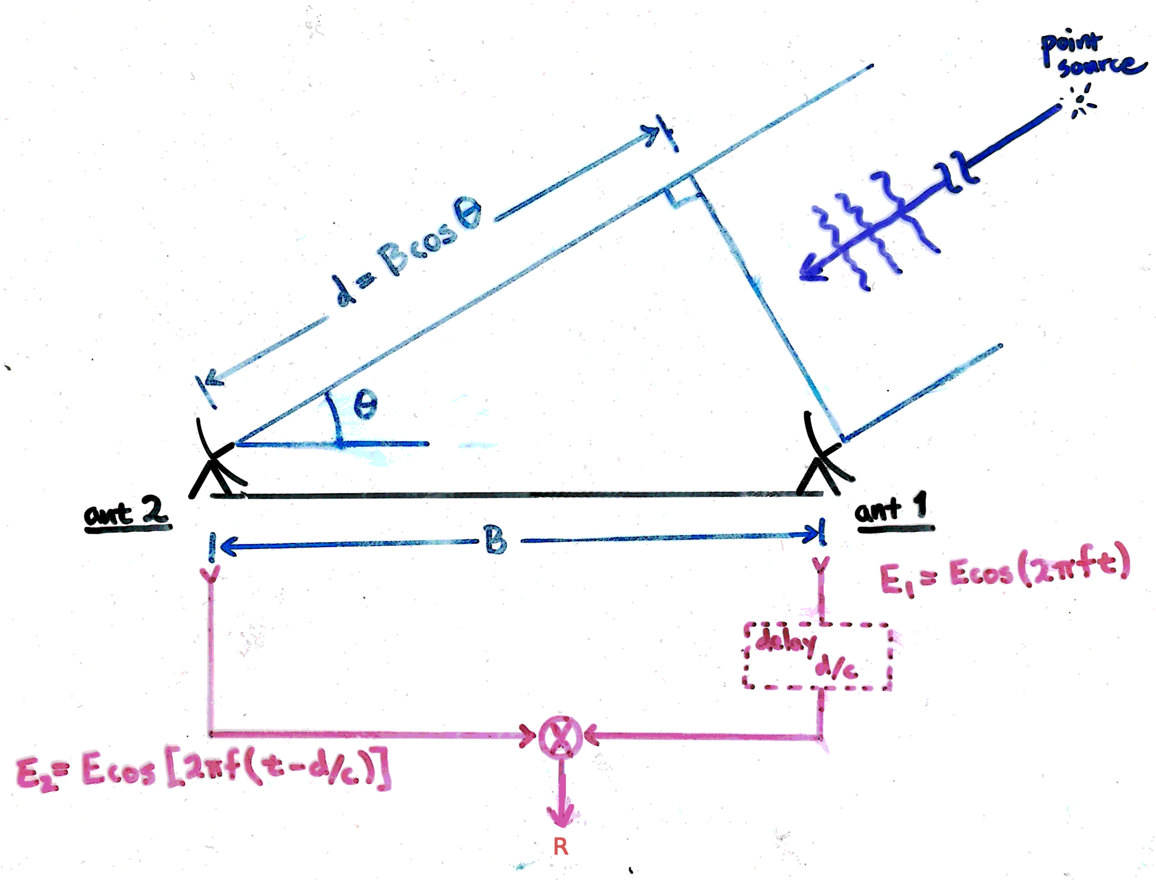

Figure 1: Geometry for interferometer observing a point source.

where

$$\phi = 2\pi f d/c \tag{3}$$

is the phase difference between the two signals. Since

$$f = c/\lambda \tag{4}$$

where λ is the wavelength, we may write the phase difference ϕ as

$$\phi = 2\pi d/\lambda. \tag{5}$$

The signals from the two antennas are multiplied together in a correlator:

$$E_1 E_2 = E\cos(2\pi f t - \phi) \cdot E\cos(2\pi f t) = (E^2/2)[\cos(4\pi f t - \phi) + \cos\phi]. \tag{6}$$

The high frequency term is filtered out and we are left with:

$$R = (E^2/2)\cos\phi = I\cos\phi. \tag{7}$$

Both the magnitude I, and the phase ϕ, of R are measured. The magnitude depends on the brightness, or intensity, of the source. The phase contains the positional information. The length d in fig. 1 can be written $B\cos\theta$. Therefore the phase is:

$$\phi = \frac{2\pi}{\lambda} B\cos\theta. \tag{8}$$

Now we have a method of determining θ, the position of the point source, from the known quantities ϕ and B.

Note that as the earth rotates, θ will change, and if B/λ is large (as it usually is), θ will be changing quite rapidly. Since this is inconvenient for electronic sampling, a variable delay is inserted between **antenna 1** (nearest to the source) and the correlator. This delay is varied at the sidereal rate of the source position to bring the phase at the center of the field of view to zero. This is known as "fringe stopping" and the point of zero phase is known as the *phase tracking center*. When this is done, the response of the correlator to a source at the phase tracking center is simple I, the source intensity.

A useful analogy is now evident. Fringe stopping has the effect of setting θ to $\pi/2$, and the baseline length to $d = B\sin\theta$. The interferometer thus acts as if the source were at the zenith and the separation were d. Note the similarity to Young's double slit experiment.

Now we consider extended sources and imaging. A complex source can be considered as a collection of point sources of varying position and intensity. We pick some source at or near the source of interest and use it as the phase tracking center. If a point on the source is $\delta\theta$ away from the phase center, then the new path difference is (see fig 2):

$$d' = B\cos(\theta + \delta\theta). \tag{9}$$

This means that the phase measured will be

$$\phi = \frac{2\pi}{\lambda} B\cos(\theta + \delta\theta). \tag{10}$$

Recall from calculus that $f(x + \delta x) = f(x) + \delta x f'(x)$, so

$$\phi = \frac{2\pi}{\lambda} B(\cos\theta + \delta\theta\sin\theta). \tag{11}$$

If we now apply the proper delay to set the phase tracking center, effectively setting $\theta = \pi/2$, then

$$\phi \approx \left(\frac{2\pi}{\lambda}\right) B\delta\theta, \tag{12}$$

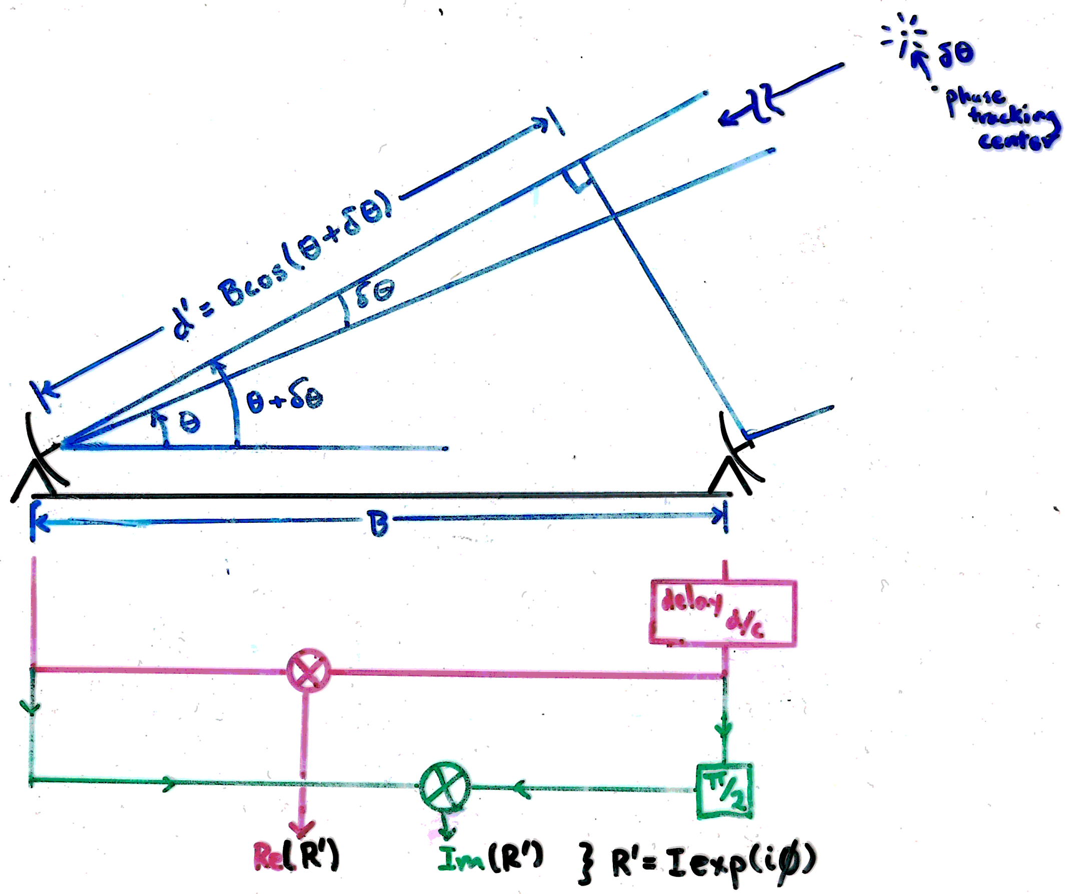

Figure 2: Geometry for an extended source.

which is the visibility phase. The response of the correlator is then

$$R = I\cos\left[\left(\frac{2\pi}{\lambda}\right)B\delta\theta\right]. \tag{13}$$

We now have a problem. Note that we have no way of telling whether $\delta\theta$, and thus ϕ, is positive or negative, since the cosine is an even function. Look at equation 6. We can shift the phase of E_2 by $\pi/2$ using a quarter cycle delay or a Hilbert Transform filter. This converts the cosine into a sine, and we have

$$E_1E_2 = E\cos(2\pi ft)E\sin(2\pi ft) \tag{14}$$
$$= I[\sin(4\pi ft+\phi)+\sin(\phi)]. \tag{15}$$

This gives us the sign of ϕ. Again filtering out the high frequency term, wer are left with $I\sin\phi$. Now we can define

$$R' = I[\cos\phi + i\sin\phi] = Ie^{i\phi}. \tag{16}$$

We define R' as a complex number since it is a compact and mathematically convenient representation for a quantity which possesses an amplitude (intensity) and a phase, both of which can be manipulated and measured electronically. R' is known as the *complex correlation,* or *visibility.*

This is the key to interferometry. The response of the (now complex) correlator to a complicated pattern of point sources will be the superposition of many sine and cosine functions of varying frequency,[1] and amplitude, which can be represented by complex numbers. The fourier transform of this

[1]N. B. The frequency of the correlator output is determined by the phase, which is dependent on the position of the source relative to the phase tracking center. See eq. 13.

superposition will yield a "spectrum" of intensities at various complex phases, and by equations 13—16 complex phase corresponds directly to position. Thus we can produce an image (one-dimensional in this case) of part of the sky.

Note than ϕ is limited to $\pm\pi/2$ by sign ambiguities with the trig functions. At any particular receiver frequency f we are thus limited in the range of angular separations we can sample for a particular baseline. This is why radio interferometers usually consist of many antennas, giving many baseline pairs. In addition, an array on the earth can take advantage of the earth's rotation to increase the range of baselines it can sample. Long observations allow the projected baseline lengths to change significantly with the earth's rotation.

The above explanation can easily be extended to two-dimensional arrays, allowing us to make actual images of the radio sky. We must then consider the baselines and positions as vectors, and use two-dimensional fourier transforms.