More refraction, Huygens, Fermat

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Basic rules to remember

If n_i is the incident medium, and n_t is the refracting (transmitted) medium, then

$n_t > n_i$ the rays bend *toward* the normal (perpendicular).

$n_t < n_i$ the rays bedn *away from* the normal.

For normal materials, the frequency remains constant, $\nu_t = \nu_i$, which means that the wavelength in a material of index n is

$$\lambda = \frac{\lambda_0}{n},$$

where λ_0 is the vacuum wavelength.

There are nonlinear materials which can upconvert or downconvert frequencies/energies per photon—used in modern photonic devices. (See p. 103)

Huygen's Principle

Every point on a propagating wavefront serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.

If the propagating wave has a frequency ν, and is transmitted through the medium at a speed v_t, then the secondary wavelets have that same frequency and speed.

Later refined mathematically by Fresnel and Kirchhoff to include interference and eliminate the "back" wavelets.

Variational principles—Hero, Fermat

Hero of Alexandria 200 B.C. (?) the path taken on reflection is the shortest possible one.

Fermat, 1657 the actual path taken by a beam of light is the one that is traversed in the least time.

Modern Fermat Principle a light ray follows a path that is stationary with respect to small variations of that path.

Fermat stimulated a great deal of work to found mechanics on similar principles, leading to the *Principle of Least Action* (Hamilton) and many other variational formulations.

Optical path length

Suppose you have a material of m layers of differing n (fig 4.30). If the s_i are the distances the beam travels in each layer, then the total time is

$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \cdots + \frac{s_m}{v_m}$$

or

$$t = \sum_{i=1}^{m} s_i/v_i$$

where v_i is the speed in each layer. With $n_i = c/v_i$ this is

$$t = \frac{1}{c} \sum_{i=1}^{m} n_i s_i.$$

The sum is called the **optical path length.**

$$OPL = \sum_{i=1}^{m} n_i s_i.$$

Most generally, for a medium of varying index with position $n(s)$, in going from point S to P:

$$OPL = \int_S^P n(s)\, ds$$

the OPL corresponds to a vacuum distance equivalent to that produced in the medium of $n(s)$. The integral is *along a route* from S to P.

Fermat Principle: in going from S to P, light traverses the route with lowest OPL.

Stationary paths—Modern Fermat

A path in which the OPL changes very little compared to nearby paths is *stationary* with respect to OPL.

For a 1–D example, a stationary value of x for $f(x)$ is one where $df/dx = 0$, a minimum, maximum or inflection point.

The OPL does **not** have to be a minimum to be stationary. See the example of elliptical mirrors on pgs. 110–111.