Quantum Theory of Metals

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Electron energies in metals

To understand thermal and electrical conduction we must realize that E levels up to the Fermi Energy E_F are fully occupied. (See fig. 12-14 of the "Fermi sphere".)

Substitute v_F for v_{RMS} into the classical expressions for σ, K.

Must also use a heat capacity C appropriate for electrons. Only a small fraction of the electrons are available to store thermal E, since most are locked in by the "Fermi sea".

Making these substitutions (eqns. 12.24–12.26) leads to

$$\frac{K}{\sigma T} = \frac{\pi^2 k_B^2}{3e^2}$$
$$= 2.45 \times 10^{-8}\ \mathrm{W} \cdot \Omega/\mathrm{K}^2,$$

which is in excellent agreement with the measured values (Table 12.7).

Quantum Mean Free Path

If we estimate the mean free path of electrons using v_F and experimental σ,

$$L = \frac{m_e v_F \sigma}{n e^2},$$

we get a value that is *many times* the spacing between metal atoms.

Quantum calculations show that electrons can travel through a perfect lattice for long distances,as the evenly spaced atoms allow electron wave to pass without scattering. Resistance is due to thermal displacements and imperfections of the lattice.

Phonons

Vibrations of the lattice are of course quantized too, with energy $\hbar\omega$. These are **phonons** and obey Bose-Einstein statistics. Just like photons, there are more of them produced thermally at high T:

$$n_p \propto \frac{k_B T}{\hbar\omega}.$$

So the number of scatterers of electrons, and thus the resistivity ρ, is proportional to T.

Imperfections in the crystal lattice will produce a contribution to ρ that is not dependent on T. This is **Matthiesen's rule**.

Problems for this section

12.10, 12.13—Calculating τ, L etc. for Silver both classically and using Fermi energy E_F.