Quantum Cascade Laser Theory
Output Power

Dr. Christopher S. Baird, University of Massachusetts Lowell

1.0 Introduction

Finding the laser output power at a certain frequency is the end purpose of the QCL code. The photon population at a certain frequency is found self-consistently using the rate equations. With the photon population known, it is straight-forward to find the output power from the population and the device characteristics. In theory, we only need to calculate the output power from the population at the very end of the calculations, after all self-consistent loops have converged. In practice, it is helpful to the user to have an order-of-magnitude estimate of the optical power before the loops converge. For this reason, the output power calculation are placed inside the loops and calculated every time a new photon population as found. Considering the fact that the output power calculations are very simple and quick to perform, placing them inside the loops does not incur any substantial run-time degradation.

The QCL code does not ask the user to specify in advance which transition is the laser transition because the user's knowledge is often faulty. Rather, it is the task of the code to determine the laser transition. In order to do this, the rate equations calculate the photon populations for all possible transitions. Further, the output powers are calculated for all possible transitions. The laser transition is then identified as the one with the highest output power.

Large photon populations are likely for transitions with frequencies below 1 THz in addition to the frequency of interest. However, such laser frequencies do not occur experimentally. The reason for this is that traditional QCL waveguides do not support modes at frequencies below 1 THz, as the loss is too high and the confinement too low. It is therefore imperative for the QCL code to properly calculate the waveguide effects at low frequencies and apply them as part of the photon population and output power calculations in order to realistically damp down the high power response of low frequency transitions. Slight inaccuracies in the waveguide calculations can lead to false reports of high-power, low-frequency lasing.

2.0 Derivation

The laser radiated output power P_{out} at a certain frequency is the energy emitted per unit time out of the front facet.
The radiated power at a frequency equals the total number of photons M at that frequency emitted per unit time, times the energy per photon E:

$$P_{\text{out}}(\omega) = M\,E$$

$$P_{\text{out}}(\omega)=M\hbar\omega$$

The number of photons of a certain frequency being emitted per unit time out the front surface of a laser must equal the total number of photons present m_{tot} at that frequency inside the laser cavity times the rate W_m at which a single photon is emitted out of the front surface mirror,

$$M=m_{tot}W_m$$

so that

$$P_{\text{out}}(\omega)=m_{tot}W_m\hbar\omega \quad .$$

The total number of photons at a frequency equals the volume of the photon region V_p times the photon population density m

$$m_{tot}=V_p m$$

so that

$$P_{\text{out}}(\omega)=V_p m W_m\hbar\omega \quad .$$

The volume of the photon cavity is defined as the volume of the active region V divided by the confinement factor Γ.

$$V_p=\frac{V}{\Gamma}$$

so that

$$P_{\text{out}}(\omega)=\frac{V m W_m\hbar\omega}{\Gamma} \quad .$$

2.1 Mirror Effects

One photon can be thought of as bouncing back and forth between the two end mirrors of the cavity. Every time it hits the front surface mirror, it has a certain chance of being emitted based on the mirror's reflectivity. The effects of the mirrors reflectivity can be averaged over one round trip of the photon. The rate W at which a single photon is emitted out the front surface equals the total probability γ_2 of a photon being emitted out of the second mirror during one round trip divided by the time it takes the photon to traverse one round trip:

$$W_m = \frac{\gamma_2}{\Delta t}$$

The time Δt it takes to make a round trip is just the distance traveled in one round trip (twice the length of the cavity l) divided by the velocity v of the light in the material:

$$\Delta t = \frac{2l}{v}$$

$$\Delta t = \frac{2ln}{c}$$

so that

$$W_m = \frac{\gamma_2 c}{2ln} \quad .$$

Define the loss per unit length α_{M2} due to emission out mirror two as the total loss γ_2 in one round trip divided by the length of one round trip $2l$,

$$\alpha_{M2} = \frac{\gamma_2}{2l}$$

so that

$$W_m = \alpha_{M2} \frac{c}{n} \quad .$$

The mirror loss α_{M2} and the effective index of refraction n were found in the waveguide calculations.

2.2 Final Equation

Plugging in the mirror loss rate into the output power equation, we find:

$$\boxed{P_{\text{out}}(\omega) = \frac{V\, m\, \alpha_{M2}\, c\, \hbar\, \omega}{n\, \Gamma}}$$

This defines a power spectrum as a function of frequency. Typically there is only one frequency with non-negligible output power. However, the power is calculated using this equation for all possible transitions to ensure completeness. Note that the photon population *m*, the effective index of refraction *n*, and the confinement factor Γ are all frequency dependent and must be calculated separately for each possible transition.

If the total power radiated into all frequencies is desired, we can simply sum the individual frequencies.

$$\boxed{P_{\text{out}}^{\text{tot}} = \sum_{\omega} P_{\text{out}}(\omega)}$$