Jackson 7.3 Homework Problem Solution

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PROBLEM:

Two plane semi-infinite slabs of the same uniform, isotropic, nonpermeable, lossless dielectric with index of refraction n are parallel and separated by an air gap ($n = 1$) of width d. A plane electromagnetic wave of frequency ω in free space is incident on the gap from one of the slabs with angle of incidence i. For linear polarization *both* parallel to *and* perpendicular to the plane of incidence,

(a) calculate the ratio of power transmitted into the second slab to the incident power and the ratio of reflected to incident power;

(b) for i greater than the critical angle for total internal reflection, sketch the ratio of transmitted power to incident power as a function of d measured in units of wavelength in the gap.

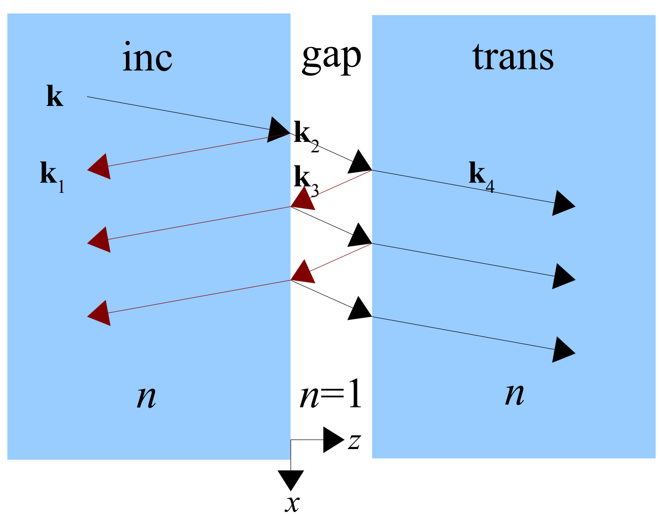
SOLUTION:

This problem is very interesting because it contains the fundamental physics behind etalons, interferometers, and Fabry-Perot cavities. There are three separate regions of uniform material, so we set up different electric fields in each and then relate them using boundary conditions. Because the incident wave is a plane wave, and the interfaces are flat, we assume the fields in all regions take on the form of plane waves. Let us call the incident material region "inc", the air gap region "gap", and the last slab region "trans". Place the interface between the incident slab and the gap at $z = 0$ and the other interface at $z = d$. In the incident slab, there is a forward-going wave (the incident wave) and a backward-going wave (the sum of all reflected waves). In the gap there is also a forward-going wave (the sum of all forward-reflected waves) and a backward-going wave (the sum of all backward-reflected waves). In the transmitted slab there is only a forward-going wave (the sum of all transmitted waves). Note that all materials are lossless so that n and k are real-valued. The waves are all assumed to have linear polarization:

$$\mathbf{E}_{\text{inc}} = \hat{\boldsymbol{\epsilon}}_0 E_0 e^{i\left(\frac{n}{c}\omega \hat{\mathbf{k}}\cdot\mathbf{x} - \omega t\right)} + \hat{\boldsymbol{\epsilon}}_1 E_1 e^{i\left(\frac{n}{c}\omega_1 \hat{\mathbf{k}}_1\cdot\mathbf{x} - \omega_1 t\right)}$$

$$\mathbf{E}_{\text{gap}} = \hat{\boldsymbol{\epsilon}}_2 E_2 e^{i\left(\frac{1}{c}\omega_2 \hat{\mathbf{k}}_2\cdot\mathbf{x} - \omega_2 t\right)} + \hat{\boldsymbol{\epsilon}}_3 E_3 e^{i\left(\frac{1}{c}\omega_3 \hat{\mathbf{k}}_3\cdot\mathbf{x} - \omega_3 t\right)}$$

$$\mathbf{E}_{\text{trans}} = \hat{\boldsymbol{\epsilon}}_4 E_4 e^{i\left(\frac{n}{c}\omega_4 \hat{\mathbf{k}}_4\cdot(\mathbf{x}-\mathbf{d}) - \omega_4 t\right)}$$



Note that the transmitted field in the last slab was shifted a distance d in the z direction to account for the fact that it is created at $z = d$ and not $z = 0$, despite being defined as relative to $z = 0$.

The boundary conditions must hold for all time and all points on the boundary. This means that the exponentials must match at $z = 0$ and $z = d$, leading to:

$$\left[e^{i(\frac{n}{c}\omega\hat{\mathbf{k}}\cdot\mathbf{x}-\omega t)}=e^{i(\frac{n}{c}\omega_1\hat{\mathbf{k}}_1\cdot\mathbf{x}-\omega_1 t)}=e^{i(\frac{1}{c}\omega_2\hat{\mathbf{k}}_2\cdot\mathbf{x}-\omega_2 t)}=e^{i(\frac{1}{c}\omega_3\hat{\mathbf{k}}_3\cdot\mathbf{x}-\omega_3 t)}\right]_{z=0} \quad \text{and}$$

$$\left[e^{i(\frac{1}{c}\omega_2\hat{\mathbf{k}}_2\cdot\mathbf{x}-\omega_2 t)}=e^{i(\frac{1}{c}\omega_3\hat{\mathbf{k}}_3\cdot\mathbf{x}-\omega_3 t)}=e^{i(\frac{n}{c}\omega_4\hat{\mathbf{k}}_4\cdot(\mathbf{x}-\mathbf{d})-\omega_4 t)}\right]_{z=d}$$

These two sets of equations must be true for all times t, so that the coefficients of t must match independently, leading to: $\boxed{\omega=\omega_1=\omega_2=\omega_3=\omega_4}$. With the time components all canceled out, these two sets of equations now become:

$$\left[n\hat{\mathbf{k}}\cdot\mathbf{x}=n\hat{\mathbf{k}}_1\cdot\mathbf{x}=\hat{\mathbf{k}}_2\cdot\mathbf{x}=\hat{\mathbf{k}}_3\cdot\mathbf{x}\right]_{z=0} \quad \text{and} \quad \left[\hat{\mathbf{k}}_2\cdot\mathbf{x}=\hat{\mathbf{k}}_3\cdot\mathbf{x}=n\hat{\mathbf{k}}_4\cdot(\mathbf{x}-\mathbf{d})\right]_{z=d}$$

All the wave vectors lie in the same plane called the plane of incidence. We can assume we have aligned the plane of incidence with the *x*-*z* plane. As a result, none of the wave vectors have any y components. Expand the vectors into x and z components and define these components in terms of the angles from the z axis (for example $k_x=k\sin\theta_i, k_z=k\cos\theta_i$). Evaluate at $z = 0$ and $z = d$. Note that evaluating at specific z locations reduces the z-component equations down to just a bunch of constants. They have no meaning at this point because we can always suck a constant phase factor into the remaining undetermined coefficients E_0, E_1, etc. Because of the lack of meaningful information, we completely drop the z components. All that remains is the x components:

$$n\sin\theta_i=n\sin\theta_r=\sin\theta_{g,i}=\sin\theta_{g,r} \quad \text{and} \quad \sin\theta_{g,i}=\sin\theta_{g,r}=n\sin\theta_{t,i}$$

This leads to:

$$\boxed{\theta_i=\theta_r \ , \ \theta_{g,i}=\theta_{g,r} \ , \ \theta_i=\theta_{t,i} \ , \ n\sin\theta_i=\sin\theta_{g,i} \ , \ \sin\theta_{g,i}=n\sin\theta_{t,i} \ ,}$$

These are just the law of reflection and Snell's law applied at both interfaces. With these applied, our fields become:

$$\mathbf{E}_{\text{inc}}=\hat{\boldsymbol{\epsilon}}_0 E_0 e^{i(\frac{n}{c}\omega(\sin\theta_i x+\cos\theta_i z)-\omega t)}+\hat{\boldsymbol{\epsilon}}_1 E_1 e^{i(\frac{n}{c}\omega(\sin\theta_i x-\cos\theta_i z)-\omega t)}$$

$$\mathbf{E}_{\text{gap}}=\hat{\boldsymbol{\epsilon}}_2 E_2 e^{i(\frac{1}{c}\omega(n\sin\theta_i x+\sqrt{1-n^2\sin^2\theta_i}z)-\omega t)}+\hat{\boldsymbol{\epsilon}}_3 E_3 e^{i(\frac{1}{c}\omega(n\sin\theta_i x-\sqrt{1-n^2\sin^2\theta_i}z)-\omega t)}$$

$$\mathbf{E}_{\text{trans}}=\hat{\boldsymbol{\epsilon}}_4 E_4 e^{i(\frac{n}{c}\omega(\sin\theta_i x+\cos\theta_i(z-d))-\omega t)}$$

These are the most explicit forms of these equations. Aside from the currently unknown field strengths, E_1, E_2, etc, everything is defined in terms of the known index of refraction n and angle of incidence θ_i. For ease of future calculations, however, let us simplify these equations. Note that because we are dealing with plane waves and flat interfaces, we can work with all these fields at the lateral position $x = 0$ without any loss of generality. In addition, we can evaluate the fields at time $t = 0$ without any loss of generality. Lastly, we use the shorthand notation $\cos\theta_{g,i}=\sqrt{1-n^2\sin^2\theta_i}$. With these simplifications, the fields become:

$$\mathbf{E}_{\text{inc}}=\hat{\boldsymbol{\epsilon}}_0 E_0 e^{i\frac{n}{c}\omega\cos\theta_i z}+\hat{\boldsymbol{\epsilon}}_1 E_1 e^{-i\frac{n}{c}\omega\cos\theta_i z}$$

$$\mathbf{E}_{\text{gap}}=\hat{\boldsymbol{\epsilon}}_2 E_2 e^{i\frac{1}{c}\omega\cos\theta_{g,i} z}+\hat{\boldsymbol{\epsilon}}_3 E_3 e^{-i\frac{1}{c}\omega\cos\theta_{g,i} z}$$

$$\mathbf{E}_{\text{trans}}=\hat{\boldsymbol{\epsilon}}_4 E_4 e^{i\frac{n}{c}\omega\cos\theta_i (z-d)} \qquad \text{where } \cos\theta_{g,i}=\sqrt{1-n^2\sin^2\theta_i}$$

All that is left is to find the field magnitudes by applying boundary conditions. To do this, we need to approach both possible polarization cases separately.

For polarization *perpendicular* to the plane of incidence, all the polarization vectors point in the positive *y* direction. Because of $\mathbf{B}=(n/c)\hat{\mathbf{k}}\times\mathbf{E}$, this tells us that all of the forward going waves have **B** fields pointing in the negative-*x*/positive-*z* direction and all the backwards going waves have **B** pointing in the positive-*x*/positive-*z* direction (of course, they really oscillate back and forth, but at points of zero total phase they point in this direction). For this polarization, the fields become:

$$\mathbf{E}_{\text{inc}}=\hat{\mathbf{y}}\, E_0 e^{i\frac{n}{c}\omega\cos\theta_i z}+\hat{\mathbf{y}}\, E_1 e^{-i\frac{n}{c}\omega\cos\theta_i z}$$

$$\mathbf{E}_{\text{gap}}=\hat{\mathbf{y}}\, E_2 e^{i\frac{1}{c}\omega\cos\theta_{g,i} z}+\hat{\mathbf{y}}\, E_3 e^{-i\frac{1}{c}\omega\cos\theta_{g,i} z}$$

$$\mathbf{E}_{\text{trans}}=\hat{\mathbf{y}}\, E_4 e^{i\frac{n}{c}\omega\cos\theta_i (z-d)}$$

$$\mathbf{B}_{\text{inc}}=\frac{n}{c}(\sin\theta_i\hat{\mathbf{z}}-\cos\theta_i\hat{\mathbf{x}})E_0 e^{i\frac{n}{c}\omega\cos\theta_i z}+\frac{n}{c}(\sin\theta_i\hat{\mathbf{z}}+\cos\theta_i\hat{\mathbf{x}})E_1 e^{-i\frac{n}{c}\omega\cos\theta_i z}$$

$$\mathbf{B}_{\text{gap}}=\frac{1}{c}(n\sin\theta_i\hat{\mathbf{z}}-\cos\theta_{g,i}\hat{\mathbf{x}})E_2 e^{i\frac{1}{c}\omega\cos\theta_{g,i} z}+\frac{1}{c}(n\sin\theta_i\hat{\mathbf{z}}+\cos\theta_{g,i}\hat{\mathbf{x}})E_3 e^{-i\frac{1}{c}\omega\cos\theta_{g,i} z}$$

$$\mathbf{B}_{\text{trans}}=\frac{n}{c}(\sin\theta_i\mathbf{z}-\cos\theta_i\mathbf{x})E_4 e^{i\frac{n}{c}\omega\cos\theta_i (z-d)} \qquad \text{where } \cos\theta_{g,i}=\sqrt{1-n^2\sin^2\theta_i}$$

The four general boundary conditions when no charges or currents are present are:

$$[\epsilon_2\mathbf{E}_2\cdot\mathbf{n}=\epsilon_1\mathbf{E}_1\cdot\mathbf{n}]_{\text{on } S} \;,\; [\mathbf{E}_2\times\mathbf{n}=\mathbf{E}_1\times\mathbf{n}]_{\text{on } S} \;,\; [\mathbf{B}_2\cdot\mathbf{n}=\mathbf{B}_1\cdot\mathbf{n}]_{\text{on } S} \;,\; \left[\frac{1}{\mu_2}\mathbf{B}_2\times\mathbf{n}=\frac{1}{\mu_1}\mathbf{B}_1\times\mathbf{n}\right]_{\text{on } S}$$

We have two boundaries, so we have eight boundary conditions total. The materials are all non-magnetic so that $\mu_2 = \mu_1 = \mu_0$, $\epsilon=\epsilon_0$ in the gap and $\epsilon=n^2\epsilon_0$ outside the gap. The boundary conditions at both interfaces become:

(1) $\left[\mathbf{E}_{\text{gap}} \cdot \hat{\mathbf{z}} = n^2 \mathbf{E}_{\text{inc}} \cdot \hat{\mathbf{z}}\right]_{z=0}$ (2) $\left[\mathbf{E}_{\text{gap}} \times \hat{\mathbf{z}} = \mathbf{E}_{\text{inc}} \times \hat{\mathbf{z}}\right]_{z=0}$

(3) $\left[\mathbf{B}_{\text{gap}} \cdot \hat{\mathbf{z}} = \mathbf{B}_{\text{inc}} \cdot \hat{\mathbf{z}}\right]_{z=0}$ (4) $\left[\mathbf{B}_{\text{gap}} \times \hat{\mathbf{z}} = \mathbf{B}_{\text{inc}} \times \hat{\mathbf{z}}\right]_{z=0}$

(5) $\left[n^2 \mathbf{E}_{\text{trans}} \cdot \hat{\mathbf{z}} = \mathbf{E}_{\text{gap}} \cdot \hat{\mathbf{z}}\right]_{z=d}$ (6) $\left[\mathbf{E}_{\text{trans}} \times \hat{\mathbf{z}} = \mathbf{E}_{\text{gap}} \times \hat{\mathbf{z}}\right]_{z=d}$

(7) $\left[\mathbf{B}_{\text{trans}} \cdot \hat{\mathbf{z}} = \mathbf{B}_{\text{gap}} \cdot \hat{\mathbf{z}}\right]_{z=d}$ (8) $\left[\mathbf{B}_{\text{trans}} \times \hat{\mathbf{z}} = \mathbf{B}_{\text{gap}} \times \hat{\mathbf{z}}\right]_{z=d}$

Plugging in the fields into these boundary conditions, we find:

$0 = 0$ from equations (1) and (5)

$E_2 + E_3 = E_0 + E_1$ from equations (2) and (3)

$E_2 - E_3 = b(E_0 - E_1)$ from equation (4)

$E_4 = E_2 e^{ia} + E_3 e^{-ia}$ from equations (6) and (7)

$b E_4 = E_2 e^{ia} - E_3 e^{-ia}$ from equation (8)

where $b = \dfrac{n \cos\theta_i}{\sqrt{1 - n^2 \sin^2\theta_i}}$ and $a = \dfrac{1}{c}\omega d \sqrt{1 - n^2 \sin^2\theta_i}$

Considering that the incident strength E_0 is taken to be a known, we have four independent equations above in four unknowns and can therefore solve uniquely for the different field strengths. After much algebra, we solve this system of equations to find:

$$\frac{E_1}{E_0} = \frac{(1-b^2)\, i \sin a}{2b\cos a - (1+b^2)\, i \sin a}$$

$$\frac{E_2}{E_0} = \frac{b(1+b)(\cos a - i \sin a)}{2b\cos a - i(1+b^2)\sin a}$$

$$\frac{E_3}{E_0} = \frac{b(1-b)(\cos a + i \sin a)}{2b\cos a - i(1+b^2)\sin a}$$

$$\frac{E_4}{E_0} = \frac{2b}{2b\cos a - i(1+b^2)\sin a}$$

where $a = \dfrac{1}{c}\omega d \sqrt{1 - n^2 \sin^2\theta_i}$ and $b = \dfrac{n \cos\theta_i}{\sqrt{1 - n^2 \sin^2\theta_i}}$

We now seek to find the fraction of reflected and transmitted power by taking the magnitude squared of the first and last equation. We have to be careful because beyond the critical angle of total internal reflection, a and b become purely imaginary, but we can still have valid transmission via the evanescent modes. Let us approach the two cases separately. Below the critical angle, a and b are purely real-valued, leading to:

$$\boxed{R=\left|\frac{E_1}{E_0}\right|^2=\frac{(1-b^2)^2\sin^2 a}{4b^2\cos^2 a+(1+b^2)^2\sin^2 a}}$$

Perpendicular polarization, below the critical angle

$$\boxed{T=\left|\frac{E_4}{E_0}\right|^2=\frac{4b^2}{4b^2\cos^2 a+(1+b^2)^2\sin^2 a}}$$

where $a=\frac{1}{c}\omega d\sqrt{1-n^2\sin^2\theta_i}$ and $b=\frac{n\cos\theta_i}{\sqrt{1-n^2\sin^2\theta_i}}$

Greater than the critical angle, *a* and *b* become purely imaginary. Let us explicitly factor out this imaginary nature: $a=i\alpha=i\left(\frac{1}{c}\omega d\sqrt{\left|1-n^2\sin^2\theta_i\right|}\right)$ and $b=-i\beta=-i\left(\frac{n\cos\theta_i}{\sqrt{\left|1-n^2\sin^2\theta_i\right|}}\right)$. With these definitions inserted before taking the magnitude squared, and then magnitude squaring, we find:

$$\boxed{R=\left|\frac{E_1}{E_0}\right|^2=\frac{(1+\beta^2)^2\sinh^2(\alpha)}{4\beta^2\cosh^2(\alpha)+(1-\beta^2)^2\sinh^2(\alpha)}}$$

Perpendicular polarization, above the critical angle

$$\boxed{T=\left|\frac{E_4}{E_0}\right|^2=\frac{4\beta^2}{4\beta^2\cosh^2(\alpha)+(1-\beta^2)^2\sinh^2(\alpha)}}$$

where $\alpha=\frac{1}{c}\omega d\sqrt{\left|1-n^2\sin^2\theta_i\right|}$ and $\beta=\frac{n\cos\theta_i}{\sqrt{\left|1-n^2\sin^2\theta_i\right|}}$

Let us now do the same thing for the polarization where the electric fields are all in the plane of incidence. All of the forward going waves have **E** fields pointing in the negative-*x*/positive-*z* direction and all the backwards going waves have **E** pointing in the positive-*x*/positive-*z* direction. Using $\mathbf{B}=(n/c)\hat{\mathbf{k}}\times\mathbf{E}$, we find the fields for parallel polarization are:

$$\mathbf{E}_{\text{inc}}=(-\cos\theta_i\hat{\mathbf{x}}+\sin\theta_i\hat{\mathbf{z}})E_0 e^{i\frac{n}{c}\omega\cos\theta_i z}+(\cos\theta_i\hat{\mathbf{x}}+\sin\theta_i\hat{\mathbf{z}})E_1 e^{-i\frac{n}{c}\omega\cos\theta_i z}$$

$$\mathbf{E}_{\text{gap}}=(-\cos\theta_{g,i}\hat{\mathbf{x}}+n\sin\theta_i\hat{\mathbf{z}})E_2 e^{i\frac{1}{c}\omega\cos\theta_{g,i}z}+(\cos\theta_{g,i}\hat{\mathbf{x}}+n\sin\theta_i\hat{\mathbf{z}})E_3 e^{-i\frac{1}{c}\omega\cos\theta_{g,i}z}$$

$$\mathbf{E}_{\text{trans}}=(-\cos\theta_i\hat{\mathbf{x}}+\sin\theta_i\hat{\mathbf{z}})E_4 e^{i\frac{n}{c}\omega\cos\theta_i(z-d)}$$

$$\mathbf{B}_{\text{inc}}=-\hat{\mathbf{y}}(n/c)\left(E_0 e^{i\frac{n}{c}\omega\cos\theta_i z}+E_1 e^{-i\frac{n}{c}\omega\cos\theta_i z}\right)$$

$$\mathbf{B}_{\text{gap}}=-\hat{\mathbf{y}}(1/c)\left(E_2 e^{i\frac{1}{c}\omega\cos\theta_{g,i}z}+E_3 e^{-i\frac{1}{c}\omega\cos\theta_{g,i}z}\right)$$

$$\mathbf{B}_{\text{trans}}=-\hat{\mathbf{y}}(n/c)E_4 e^{i\frac{n}{c}\omega\cos\theta_i(z-d)}$$

where $\cos\theta_{g,i}=\sqrt{1-n^2\sin^2\theta_i}$

Plugging these fields into the boundary conditions shown in Equations (1)-(8), we find:

$E_2 + E_3 = n E_0 + n E_1$ from equations (1) and (4)

$n E_2 - n E_3 = b E_0 - b E_1$ from equation (2)

0 = 0 from equations (3) and (7)

$n E_4 = E_2 e^{ia} + E_3 e^{-ia}$ from equation (5) and (8)

$b E_4 = n E_2 e^{ia} - n E_3 e^{-ia}$ from equation (6)

where as usual $b = \dfrac{n\cos\theta_i}{\sqrt{1-n^2\sin^2\theta_i}}$ $\quad a = \dfrac{1}{c}\omega d\sqrt{1-n^2\sin^2\theta_i}$

We can now solve uniquely for the undetermined coefficients. After much algebra, we find:

$$\boxed{\begin{aligned}
\frac{E_1}{E_0} &= \frac{(n^4-b^2)\, i\sin a}{2n^2 b\cos a - i(n^4+b^2)\sin a}\\
\frac{E_2}{E_0} &= \frac{b\,n(n^2+b)(\cos a - i\sin a)}{2n^2 b\cos a - i(n^4+b^2)\sin a}\\
\frac{E_3}{E_0} &= \frac{b\,n(n^2-b)(\cos a + i\sin a)}{2n^2 b\cos a - i(n^4+b^2)\sin a}\\
\frac{E_4}{E_0} &= \frac{2n^2 b}{2n^2 b\cos a - i(n^4+b^2)\sin a}
\end{aligned}}$$

where $a = \dfrac{1}{c}\omega d\sqrt{1-n^2\sin^2\theta_i}$ and $b = \dfrac{n\cos\theta_i}{\sqrt{1-n^2\sin^2\theta_i}}$

For angles less than the critical angle we find:

$$\boxed{R = \left|\frac{E_1}{E_0}\right|^2 = \frac{(n^4-b^2)^2\sin^2 a}{4n^4 b^2\cos^2 a + (n^4+b^2)^2\sin^2 a}}$$

$$\boxed{T = \left|\frac{E_4}{E_0}\right|^2 = \frac{4n^4 b^2}{4n^4 b^2\cos^2 a + (n^4+b^2)^2\sin^2 a}}$$

Parallel polarization, below the critical angle

where $a = \dfrac{1}{c}\omega d\sqrt{1-n^2\sin^2\theta_i}$ and $b = \dfrac{n\cos\theta_i}{\sqrt{1-n^2\sin^2\theta_i}}$

For angles greater than the critical angle we find:

$$R = \left|\frac{E_1}{E_0}\right|^2 = \frac{(n^4+\beta^2)^2 \sinh^2(\alpha)}{4n^4\beta^2\cosh^2(\alpha)+(n^4-\beta^2)^2\sinh^2(\alpha)}$$

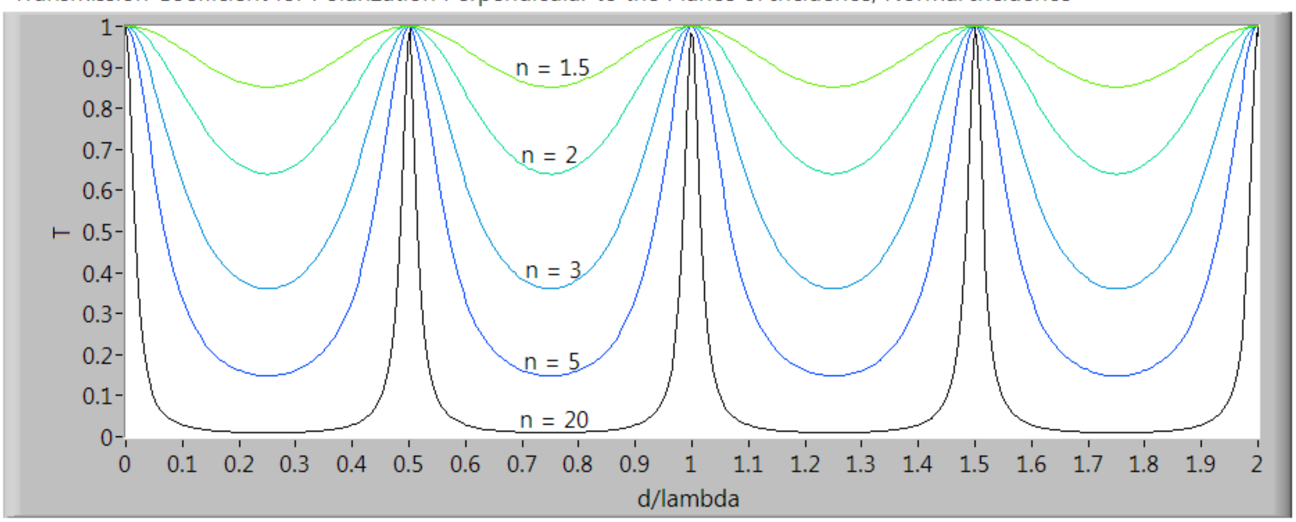
Parallel polarization, above the critical angle

$$T = \left|\frac{E_4}{E_0}\right|^2 = \frac{4n^4\beta^2}{4n^4\beta^2\cosh^2(\alpha)+(n^4-\beta^2)^2\sinh^2(\alpha)}$$

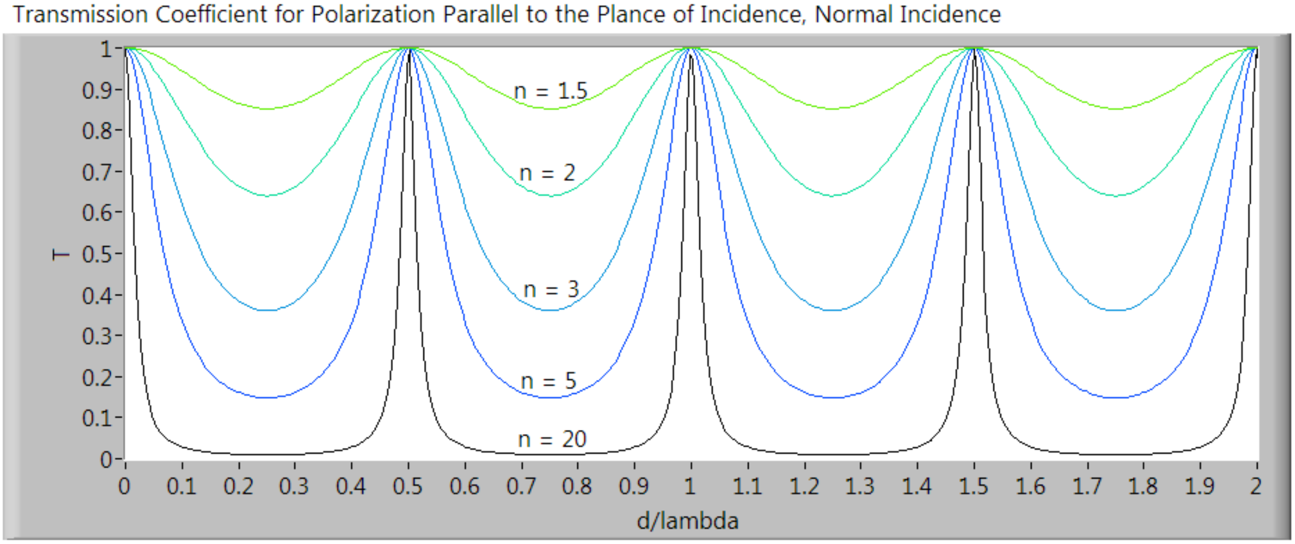
where $\alpha = \frac{1}{c}\omega d\sqrt{|1-n^2\sin^2\theta_i|}$ and $\beta = \frac{n\cos\theta_i}{\sqrt{|1-n^2\sin^2\theta_i|}}$

Let us plot all these equations to understand what they mean. We only need to plot the transmission coefficients T because the reflection coefficients are trivially related by $R = 1 - T$. The transmission coefficient represents the fraction of the incident power that makes it all the way through the system and travels out the other side. At normal incidence, which is always below the critical angle, T depends on the gap space d and the index of refraction n as shown below.

Transmission Coefficient for Polarization Perpendicular to the Plance of Incidence, Normal Incidence



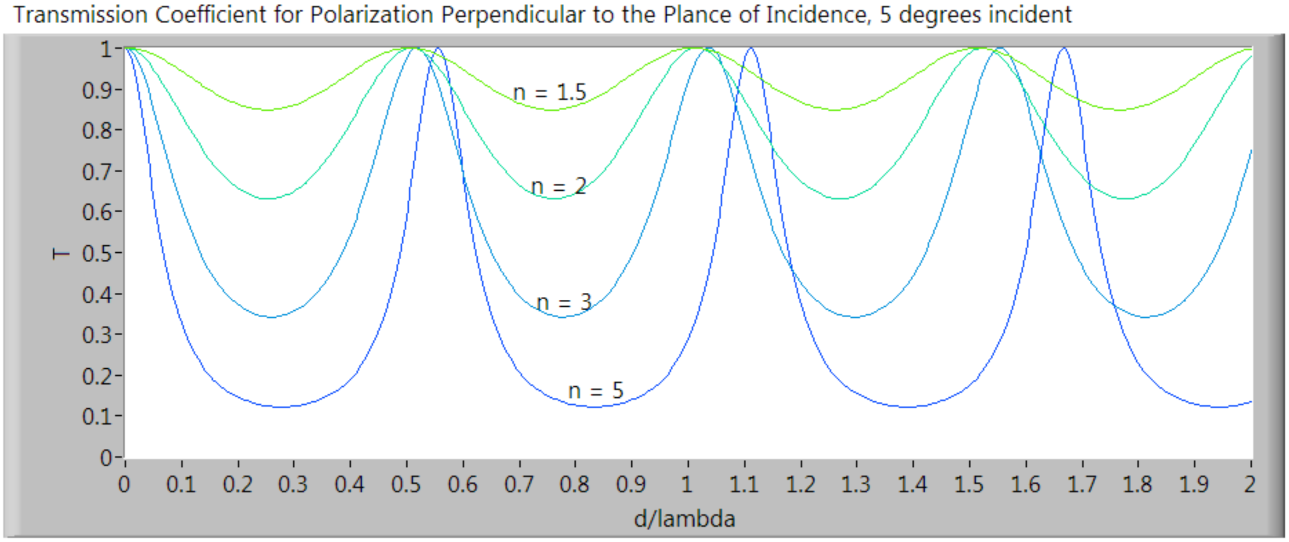
Transmission Coefficient for Polarization Parallel to the Plance of Incidence, Normal Incidence



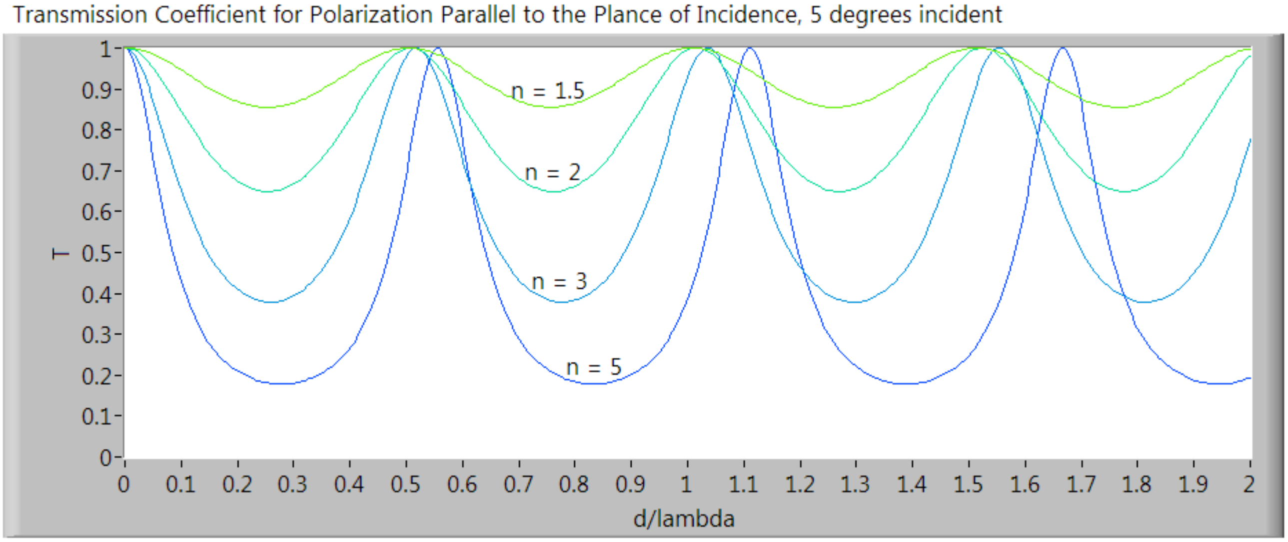
Note that due to the symmetry when we are at normal incidence, both polarizations give the same trends. The trends are periodic. When the gap spacing is close to a half-integer multiple of the wavelength of the light in the gap (when $d/\lambda = n/2$, $n = 0,1,2...$) then the transmission is high. This is because the forward reflected waves in the gap and the backward reflected wave line up so that they constructively interfere. Away from these resonance points, the backward and forward waves become unaligned and destructively interfere. As a result, the transmission drops for gap widths away from half-integer multiples of the wavelength. Also note that a higher index of refraction n leads to lower drops in transmission. This is because a higher index material in the slabs leads to greater reflection at the interfaces. As a result, the waves spend longer in the gap and participate more in destructive interference. In the limit that n approaches infinity, this device becomes a perfect monochromatic filter, only letting through one wavelength per period.

If we move away from normal incidence, the same general trends hold, but the two different polarizations behave differently now. Also, the resonance points are now not exactly at half-integer multiples of the wavelength. This is because the wave is traveling at a diagonal, so that the interference effects become more complex.

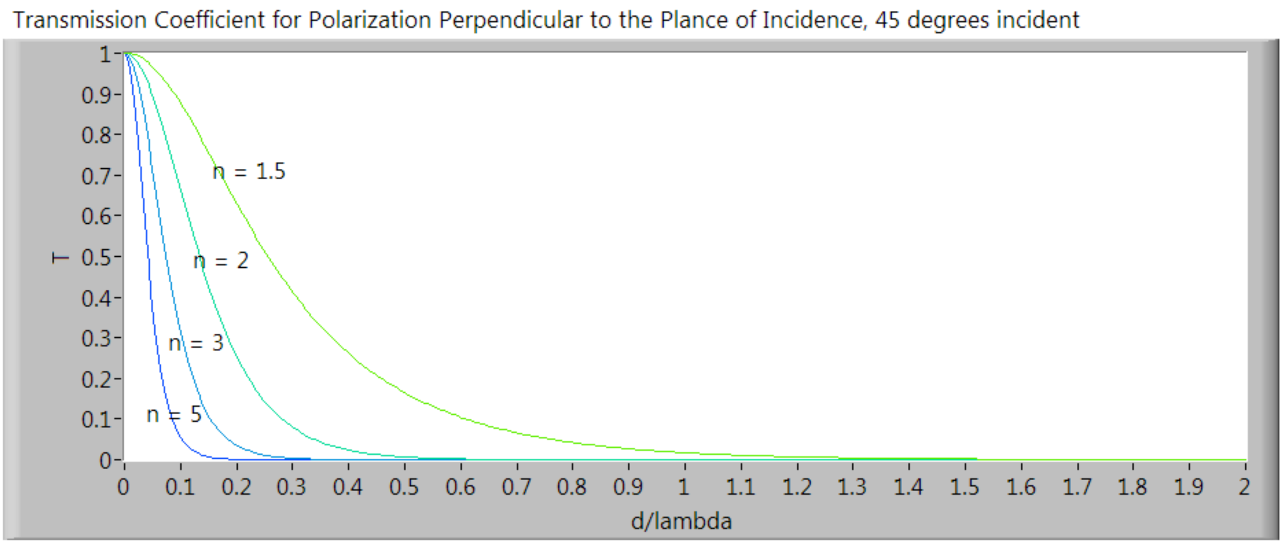
Transmission Coefficient for Polarization Perpendicular to the Plance of Incidence, 5 degrees incident

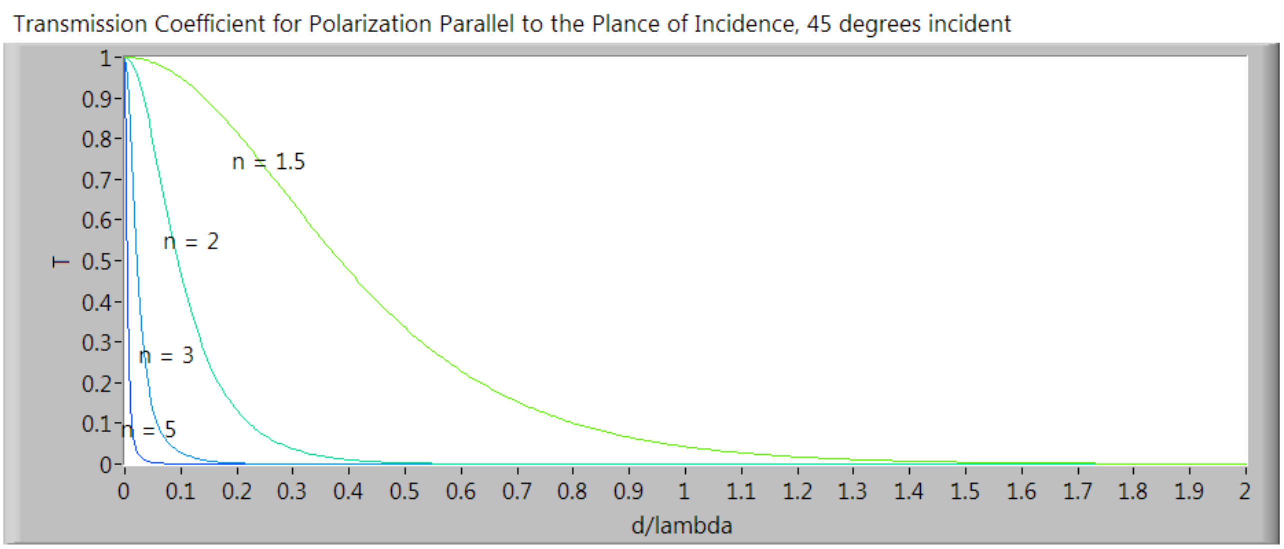


Transmission Coefficient for Polarization Parallel to the Plance of Incidence, 5 degrees incident



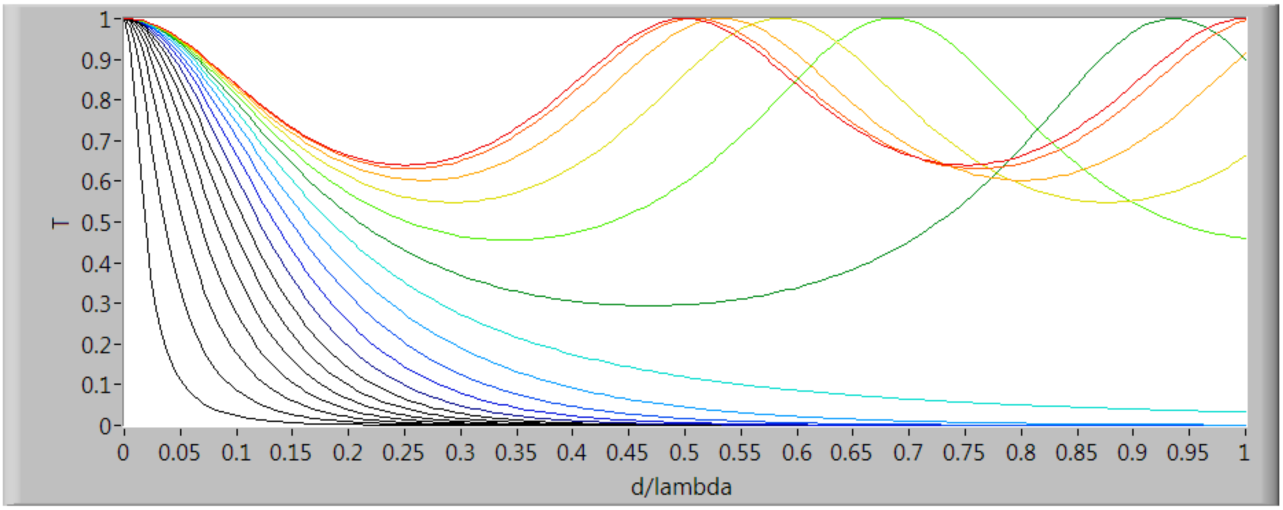
For angles of incidence greater than the critical angle of total internal reflection, transmission is only via evanescent waves in the gap, which die out exponentially. As a result, the transmission is only high for small gap widths, as shown below. The higher the index of refraction, the more strongly the incident wave is reflected, and the quicker the evanescent wave dies out. This principle is used to create tunable beam splitters. Any desired transmission coefficient can be realized by adjusting the gap width appropriately.



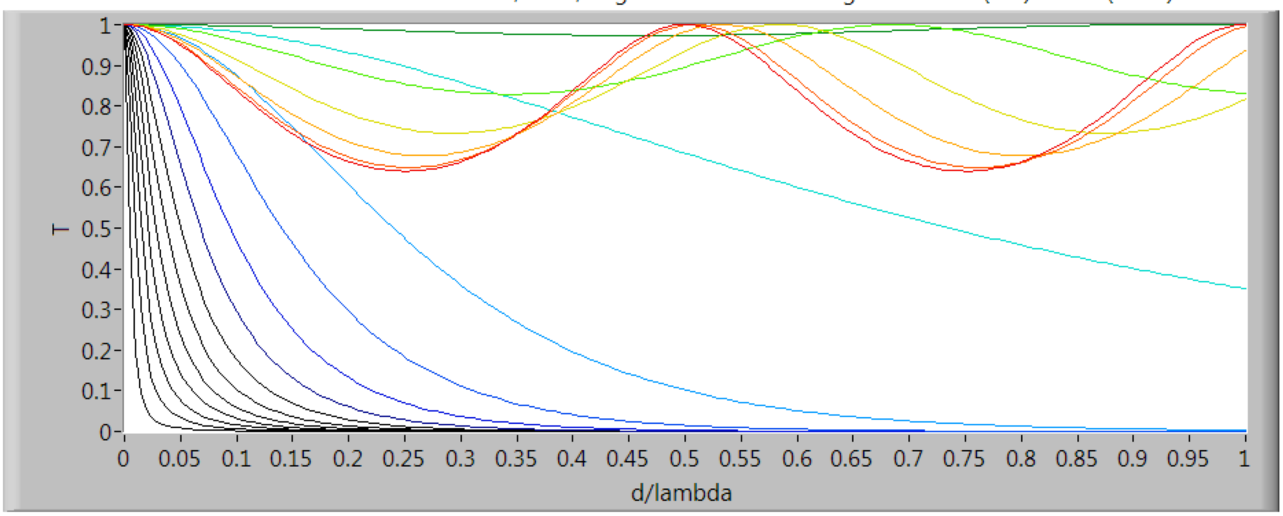


Next, we can plot many trends as a function of angle of incidence and not index of refraction to get a feel for how the transmission depends on incidence angle. The results are shown below for $n = 2$, plotted in 5° increments starting from 0° in red to 90° for the last black curve. As these plots show, the critical angle of total internal reflection divides the periodic curves from the damped curves. Also note that as the angle of incidence increases, the resonant gap widths increase. Finally note that for parallel polarization, the transmission reaches 100% for all gap widths d at a certain angle. This is Brewster's angle, as discussed in class.

Transmission Coefficient for Perpendicular Polarization, n = 2, angle of incidence from goes from 0° (red) to 90° (black)

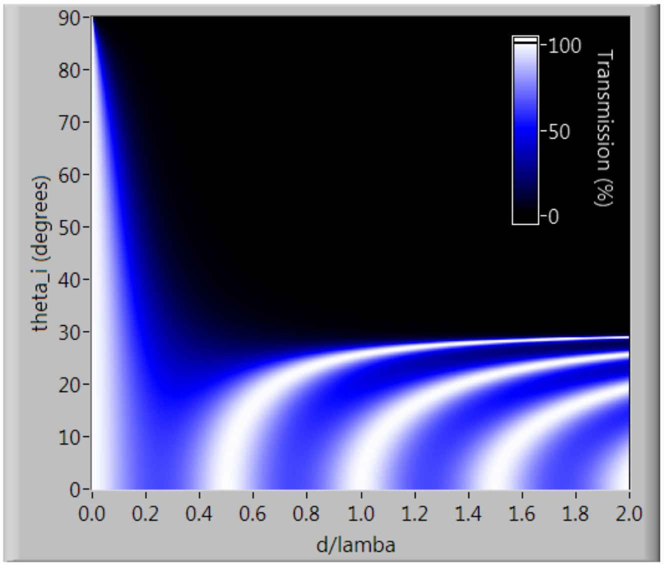


Transmission Coefficient for Parallel Polarization, n =2, angle of incidence from goes from 0° (red) to 90° (black)

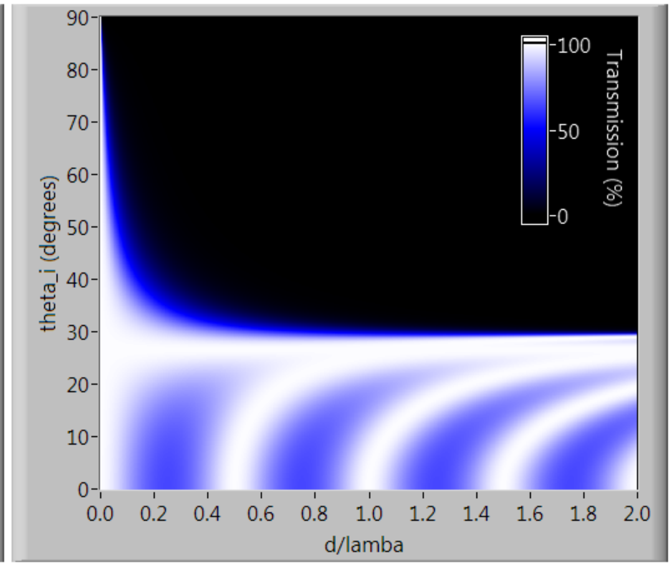


Perhaps Brewster's angle is most visible if we make an intensity plot so that the transmission can be a smooth function of both the angle of incidence and the gap width. The results are shown below. Lighter colors represent higher transmission. White represents 100% transmission and black represents 0% transmission. The x axis is the gap width d/λ and the y axis is the angle of incidence θ_i. The white streaks occur at the resonant wavelengths of the system. The resonant wavelengths start at half-integer multiples of the gap width for normal incidence, and then bend towards higher values as the angle of incidence increases. After the critical angle of total internal reflection is reached, the plot becomes black except for at very small gaps widths where tunneling is supported. Comparing the results for the perpendicular polarization and the parallel polarization, we see a bright bar of strong transmission right before the critical angle in the parallel polarization. This is Brewster's angle. Total transmission is encouraged at Brewster's angle independent of gap width, but it only occurs in one polarization. The plots below show the cases of $n = 2$ and $n = 4$. From these we see that changing the index of refraction of the materials does not change the overall pattern, but simply lowers Brewster's angle and the critical angle, as well as increasing the contrast between highs and lows.

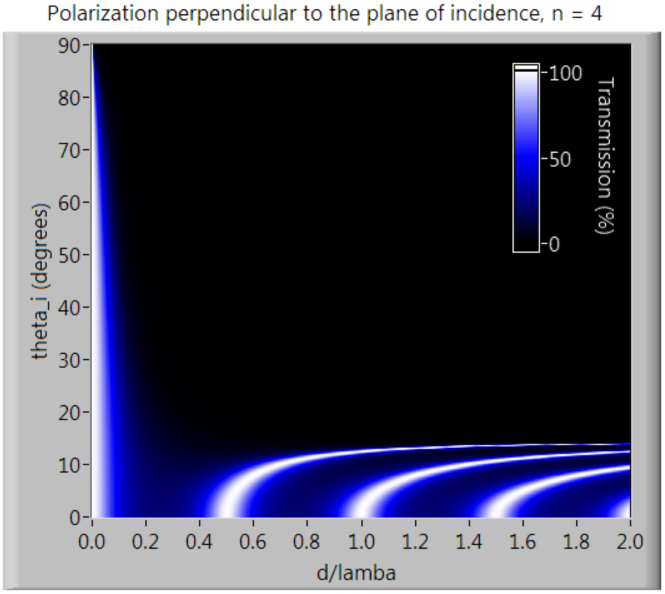
Transmission dependence of angle of incidence and gap width,
Polarization perpendicular to the plane of incidence, n = 2



Transmission dependence of angle of incidence and gap width,
Polarization parallel to the plane of incidence, n = 2



Transmission dependence of angle of incidence and gap width,
Polarization perpendicular to the plane of incidence, n = 4



Transmission dependence of angle of incidence and gap width,
Polarization parallel to the plane of incidence, n = 4