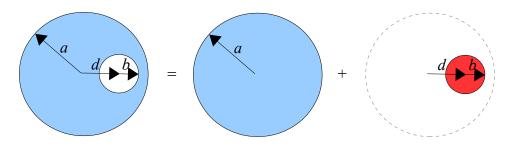## PROBLEM:

A cylindrical conductor of radius a has a hole of radius b bored parallel to, and centered a distance d from, the cylinder axis ($d + b < a$). The current density is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampere's law and principle of linear superposition to find the magnitude and direction of the magnetic-flux density in the hole.

SOLUTION:

Define the direction in which the hole is displaced from the cylinder axis as the positive x direction, and the direction coming out of the page as the positive z direction. If a uniform current density J_0 flows in the positive z direction everywhere colored blue in the leftmost image, this is equivalent to a uniform current density J_0 flowing in the positive z direction *everywhere* in the cylinder plus a uniform current density J_0 flowing in the negative z direction in the hole region.

Ampere's Law in integral form states:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot \mathbf{n} \, da$$

Draw a circular Amperian at some radius r in side the cylinder with no hole and take the line integral over this loop. Because of the symmetry, the magnetic field will be constant along this integral and can come out of the integral.

$$B_\phi \oint_C d\,l = \mu_0 J_0 \int_S da$$

$$B_\phi 2 \pi r = \mu_0 J_0 \pi r^2$$

$$\mathbf{B} = \frac{1}{2} \mu_0 J_0 r \,\hat{\boldsymbol{\phi}}$$

$$\mathbf{B} = \frac{1}{2} \mu_0 J_0 (-y \,\hat{\mathbf{i}} + x \,\hat{\mathbf{j}})$$

Using the same method, place the hole-shaped region with current flowing into the page at the origin and it produces the magnetic field:

$$\mathbf{B} = -\frac{1}{2}\mu_0 J_0(-y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$$

Now shift this region to its proper place, $x \to x - d$

$$\mathbf{B} = -\frac{1}{2}\mu_0 J_0(-y\,\hat{\mathbf{i}} + (x-d)\,\hat{\mathbf{j}})$$

And add together the two field to get the final answer:

$$\mathbf{B} = \frac{1}{2}\mu_0 J_0(-y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}) - \frac{1}{2}\mu_0 J_0(-y\,\hat{\mathbf{i}} + (x-d)\,\hat{\mathbf{j}})$$

$$\boxed{\mathbf{B} = \frac{d}{2}\mu_0 J_0\,\hat{\mathbf{j}}}$$