Jackson 2.2 Homework Problem Solution

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PROBLEM:

Using the method of images, discuss the problem of a point charge q *inside* a hollow, grounded, conducting sphere of inner radius a. Find

(a) the potential inside the sphere;

(b) the induced surface-charge density;

(c) the magnitude and direction of the force acting on q.

(d) Is there any change in the solution if the sphere is kept at a fixed potential V? If the sphere has a total charge Q on its inner and outer surfaces?

SOLUTION:

In class, we already solved the problem of a point charge outside a conducting sphere. This problem is identical except that we switch the position of the image charge and real charge.

(a) Because of the symmetry, the potential will still be the same:

$$\boxed{\Phi(\mathbf{x})=\frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\mathbf{x}-\mathbf{y}|}-\frac{1}{\left|\frac{y}{a}\mathbf{x}-\frac{a}{y}\mathbf{y}\right|}\right]}$$

(b) The surface charge density will evaluate to the same value, except that the normal is now pointing in the opposite direction, so we must add a negative sign:

$$\boxed{\sigma=\frac{q}{4\pi a^2}\left(\frac{a}{y}\right)\frac{1-\frac{a^2}{y^2}}{\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{3/2}}}$$

Note that for a real charge inside the sphere, $y < a$, so that the numerator ends up negative, so that the overall charge is still the opposite charge of q as it was for the case of the real charge outside the sphere.

(c) The force is the same as before but is directed in the opposite direction:

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left(\frac{a}{y}\right)^3\left(1-\frac{a^2}{y^2}\right)^{-2}\hat{\mathbf{y}}$$

(d) Nothing changes. If the charge Q is added to the sphere, the induced charge on the inside surface of the sphere must still be $-q$, leaving a charge $Q - q$ on the outside surface of the sphere.