Jackson 1.8 Homework Problem Solution

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PROBLEM:

(a) For the three capacitor geometries in Problem 1.6 calculate the total electrostatic energy and express it alternatively in terms of the equal and opposite charges Q and $-Q$ placed on the conductors and the potential difference between them.

(b) Sketch the energy density of the electrostatic field in each case as a function of the appropriate linear coordinate.

SOLUTION:

(a) For a simple capacitor, the total energy is given by $W = \frac{1}{2}QV$. In problem 1.6, we found the following results.

Parallel plates capacitor: $V = \frac{Q d}{A \epsilon_0}$ and $E = \frac{Q}{A \epsilon_0}$

Concentric spheres capacitor: $V = \frac{Q}{4 \pi \epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$ and $E = \frac{1}{4 \pi \epsilon_0} \frac{Q}{r^2}$

Concentric cylinders capacitor: $V = \frac{Q}{2 \pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)$ and $E = \frac{Q}{2 \pi r \epsilon_0 L}$

It is straight-forward to substitute these equations into the energy equation and find the following:

Parallel plates capacitor: $\boxed{W = \frac{Q^2 d}{2 A \epsilon_0}}$ and $\boxed{W = \frac{V^2 A \epsilon_0}{2 d}}$

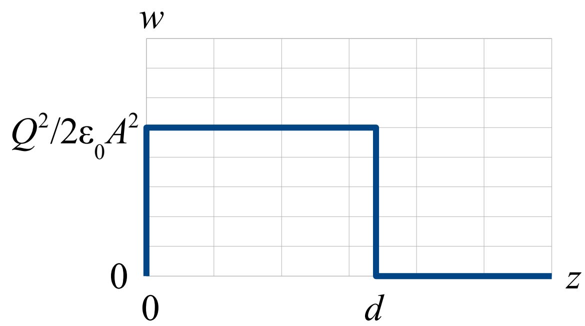
Concentric spheres capacitor: $\boxed{W = \frac{Q^2}{8 \pi \epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$ and $\boxed{W = \frac{V^2 2 \pi \epsilon_0 a b}{b - a}}$

Concentric cylinders capacitor: $\boxed{W = \frac{Q^2}{4 \pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)}$ and $\boxed{W = \frac{V^2 \pi \epsilon_0 L}{\ln(b/a)}}$

(b) The energy density is defined as $w = \frac{\epsilon_0}{2} E^2$. A simple substitution of the fields found in problem 1.6 reveals:

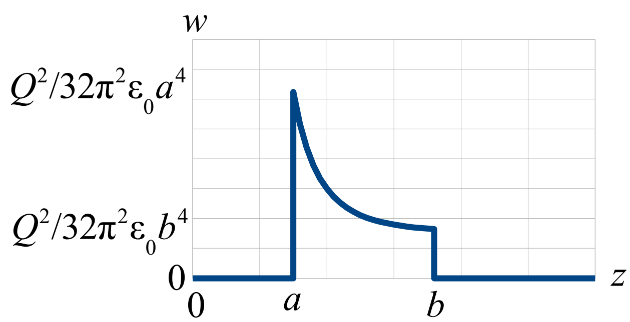
Parallel plates capacitor:

$$\boxed{w = \frac{Q^2}{2\,\epsilon_0 A^2}}$$



Concentric spheres capacitor:

$$\boxed{w = \frac{Q^2}{32\,\pi^2\,\epsilon_0} r^{-4}}$$



Concentric cylinders capacitor:

$$\boxed{w = \frac{Q^2}{8\,\pi^2\,\epsilon_0 L^2} r^{-2}}$$