Summary of Electrostatics with Dielectrics

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1. General Equations

E: Total electric field
D: Applied electric field plus interactions
P: Dielectric material polarization, leading to material response field
ρ_{total}: Total charge density
ρ: Free or excess charge density
ρ_{pol}: Polarization (bound) charge density

Gauss' Law	leads to the	Boundary Condition
$\nabla\cdot\mathbf{E}=\frac{\rho_{total}}{\epsilon_0}$		$(\mathbf{E}_2-\mathbf{E}_1)\cdot\mathbf{n}_{12}=\frac{\sigma_{total}}{\epsilon_0}$
$\nabla\cdot\mathbf{D}=\rho$		$(\mathbf{D}_2-\mathbf{D}_1)\cdot\mathbf{n}_{12}=\sigma$
$\nabla\cdot\mathbf{P}=-\rho_{pol}$		$(\mathbf{P}_2-\mathbf{P}_1)\cdot\mathbf{n}_{12}=-\sigma_{pol}$
$\nabla\times\mathbf{E}=0$		$(\mathbf{E}_2-\mathbf{E}_1)\times\mathbf{n}_{12}=0$
$\nabla\times(\mathbf{D}-\mathbf{P})=0$		$((\mathbf{D}_2-\mathbf{P}_2)-(\mathbf{D}_1-\mathbf{P}_1))\times\mathbf{n}_{12}=0$

Relations of Fields and Charges

$$\mathbf{E}=\mathbf{D}/\epsilon_0-\mathbf{P}/\epsilon_0$$

$$\rho_{total}=\rho+\rho_{pol}$$

Electrostatic Potential and Energy

$\mathbf{E}=-\nabla\Phi$ leads to $\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\int\frac{\rho_{total}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}'$

$$W=\frac{1}{2}\int\mathbf{E}\cdot\mathbf{D}\,d\mathbf{x}$$

2. Linear Isotropic Dielectric Materials

ϵ : Electric permittivity of the dielectric material

$\epsilon_r = \epsilon/\epsilon_0$: Relative electric permittivity, also called the dielectric constant

$\mathbf{D} = \epsilon\mathbf{E}$ which leads to $\mathbf{P} = \frac{\epsilon - \epsilon_0}{\epsilon}\mathbf{D}$ and $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$

- In linear materials, **E**, **D**, and **P** all have the same general field line pattern, and only differ in magnitude.

$\nabla\cdot(\epsilon\mathbf{E}) = \rho$ which in regions of uniform material, becomes $\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon}$

Boundary Conditions: $(\epsilon_2\mathbf{E}_2 - \epsilon_1\mathbf{E}_1)\cdot\mathbf{n}_{12} = \sigma$ and $(\mathbf{E}_2 - \mathbf{E}_1)\times\mathbf{n}_{12} = 0$

Electrostatic Potential and Energy

$\mathbf{E} = -\nabla\Phi$ which in regions of uniform material, leads to $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon}\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d\,\mathbf{x}'$

$W = \frac{\epsilon}{2}\int|\mathbf{E}|^2 d\,\mathbf{x}$

3. Special Cases

- Outside dielectric materials (free space), $\epsilon = \epsilon_0$ or $\epsilon_r = 1$ so that:

$\mathbf{P} = \frac{\epsilon - \epsilon_0}{\epsilon}\mathbf{D}$ becomes **P** = 0 and

$\mathbf{E} = \mathbf{D}/\epsilon_0 - \mathbf{P}/\epsilon_0$ becomes $\mathbf{E} = \frac{\mathbf{D}}{\epsilon_0}$

- Inside perfect dielectric materials, $\epsilon \to \infty$ so that:

$\mathbf{P} = \frac{\epsilon - \epsilon_0}{\epsilon}\mathbf{D}$ becomes $\mathbf{P} = \mathbf{D}$ and

$\mathbf{E} = \mathbf{D}/\epsilon_0 - \mathbf{P}/\epsilon_0$ becomes **E** = 0

Therefore, a perfect dielectric acts like a perfect conductor (only in electrostatics).

- Inside regions of uniform, linear dielectric material $\nabla\cdot\mathbf{P} = 0$ so that

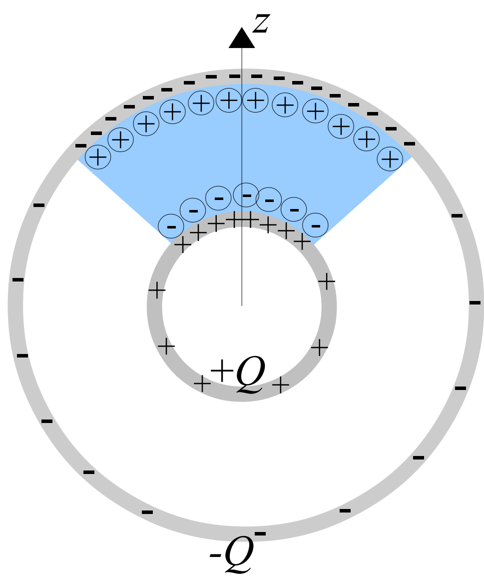
$\nabla\cdot\mathbf{P} = -\frac{\rho_{pol}}{\epsilon_0}$ becomes $\rho_{pol} = 0$ so that all polarization charge is on the surface

<u>4. Sample Diagram Problem</u>

Two conducting spherical shells are concentric. The inner conductor carries a total electric charge $+Q$ and the outer conductor carries a total electric charge $-Q$. The space between the shells, for polar angles less than β, is filled with a uniform dielectric material (shown in blue). The rest of the space between the shells is free space.

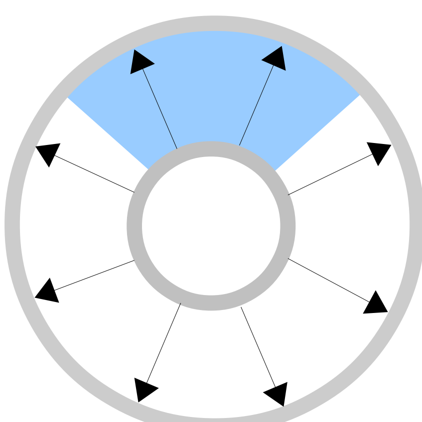
<u>Charge Densities</u>

- Free charge density shown as plus and minus signs
- Polarization charge density shown as circles containing plus and minus signs

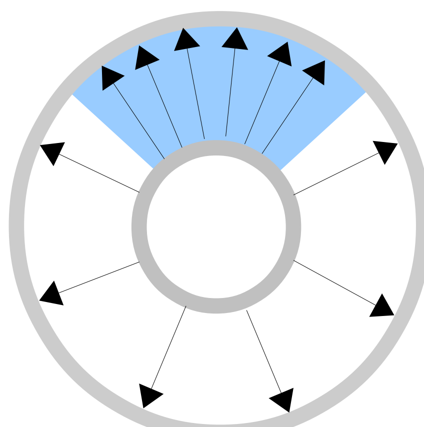


<u>Field Lines:</u> $\mathbf{E} = \mathbf{D}/\epsilon_0 - \mathbf{P}/\epsilon_0$

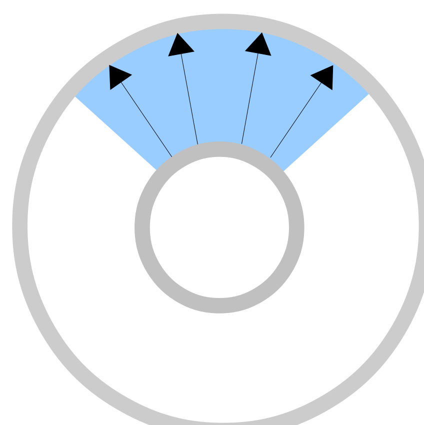
E



D



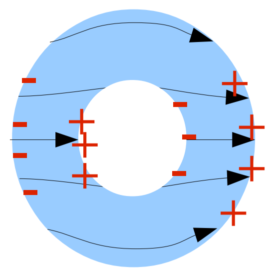
P



5. Another Sample Diagram Problem

A solid dielectric sphere has its core removed. This sphere is placed in an originally uniform electric field. There is no conducting material anywhere.

P



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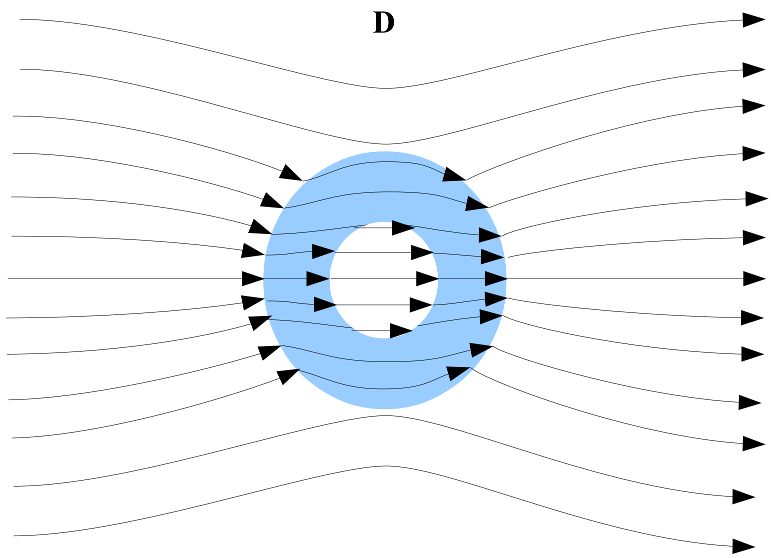

E