Magnetostatics Methods

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I. Are there permanent magnets with known $\mathbf{M}$?

Because the magnet is permanent, the fields it creates will be independent and additive to the rest of the problem. Solve for the fields due to the permanent magnets with the rest of the problem (free currents, materials, boundaries, etc.) absent. Depending on the geometry choose one method:

Option A:

1. Calculate the magnet's volume and surface magnetic charge density: $\rho_M = -\nabla \cdot \mathbf{M}$
2. Calculate the magnetic scalar potential due to the magnet: $\Phi_M = \frac{1}{4\pi} \int \frac{\rho_M(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'$
3. Calculate the fields: $\mathbf{H} = -\nabla \Phi_M$, $\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$ ($\mathbf{M}$ inside is known, $\mathbf{M}$ outside is zero)

Option B:

1. Calculate the magnet's volume and surface bound current density: $\mathbf{J}_M = \nabla \times \mathbf{M}$
2. Calculate the magnetic vector potential due to the magnet: $\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_M(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'$
3. Calculate the fields: $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$ ($\mathbf{M}$ inside is known, $\mathbf{M}$ outside is zero)

II. Are there regions in the problem with no currents, $\mathbf{J}_{\text{total}} = 0$, and linear uniform material?

1. For each region solve $\nabla^2 \Phi_M = 0$ using orthogonal functions just like in electrostatics.
2. Apply any trivial boundary conditions (finite at origin, finite at infinity, etc.). Other boundary conditions will have to wait until later.
3. Calculate the fields: $\mathbf{B} = -\nabla \Phi_M$, $\mathbf{H} = \frac{1}{\mu}\mathbf{B}$, $\mathbf{M} = \left(1 - \frac{\mu_0}{\mu}\right)\mathbf{B}$

III. Are there free currents $\mathbf{J}_{\text{free}}$ in a region of linear uniform material?

1. Calculate the magnetic vector potential due to the free currents: $\mathbf{A} = \frac{\mu}{4\pi} \int \frac{\mathbf{J}_{\text{free}}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'$
2. Calculate the fields: $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{H} = \frac{1}{\mu}\mathbf{B}$, $\mathbf{M} = \left(1 - \frac{\mu_0}{\mu}\right)\mathbf{B}$

IV. In each region, add up the fields found above.

1. In each region, add up the fields as found above due to permanent magnets, free currents, and bound surface currents (this may require the method of images to find the fields due to surface currents).
2. Apply boundary conditions to find the final solutions $(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0$, $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_{\text{free}}$