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## Magnetostatic Equations Summary

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### 1. Definitions

$\mu_0 \mathbf{H}$ : The *applied* magnetic field plus interactions.

$\mu_0 \mathbf{M}$ : The *material response* magnetic field.

$\mathbf{B}$ : The *total* magnetic field.

$\mathbf{J}$ : The *applied* current density.

$\mathbf{J}_M$ : The *material* current density

$\mathbf{J}_{\text{total}}$ : The *total* current density

### 2. Fundamental Equations to Understand and Memorize

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} \quad (\text{total field} = \text{applied field} + \text{material response field})$$

$$\mathbf{J}_{\text{total}} = \mathbf{J} + \mathbf{J}_M \quad (\text{total current density} = \text{applied current density} + \text{material current density})$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_{\text{total}} \quad (\text{total current gives rise to curling total field})$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (\text{applied current gives rise to curling applied field})$$

$$\nabla \times \mathbf{M} = \mathbf{J}_M \quad (\text{material current gives rise to curling material response field})$$

$$\nabla \times \nabla \Phi = 0 \quad (\text{math identity: if a vector field has no curl, it is the gradient of a scalar potential})$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{total magnetic field lines are not created or destroyed})$$

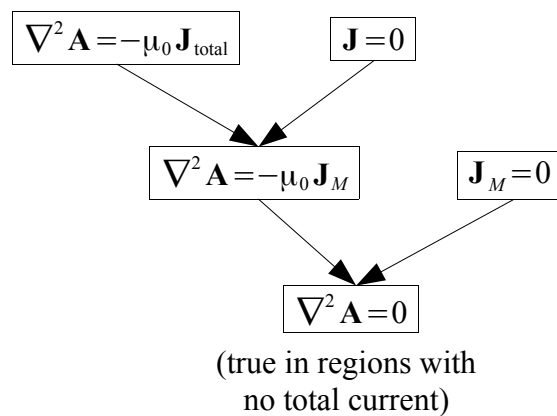
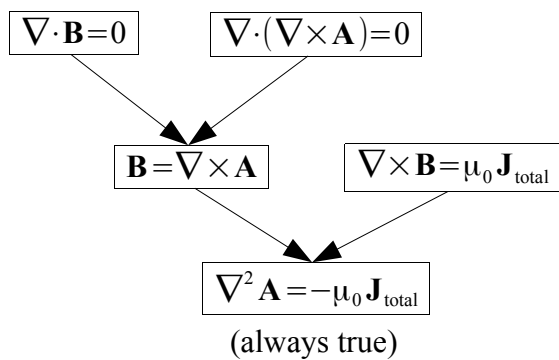
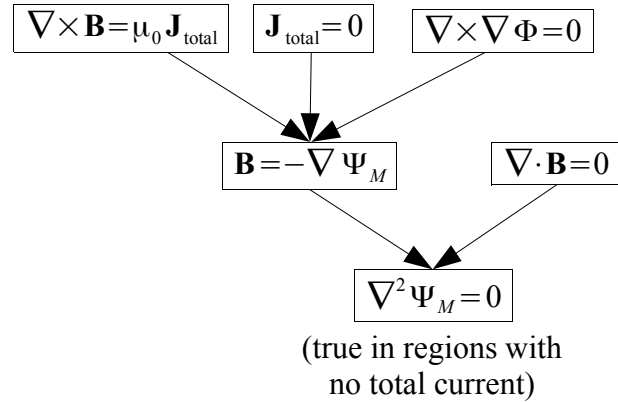
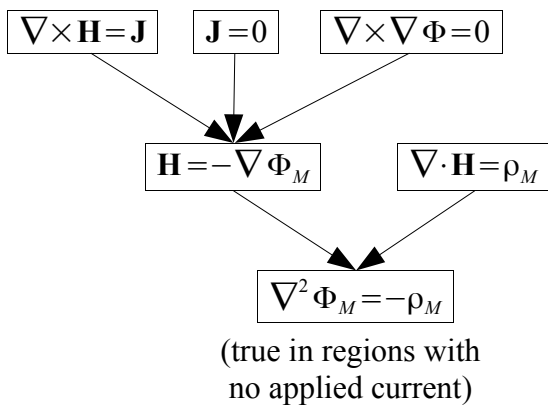
$$\nabla \cdot \mathbf{H} = \rho_M \quad (\text{positive effective magnetic charge creates } \mathbf{H} \text{ field lines, negative destroys})$$

$$\nabla \cdot \mathbf{M} = -\rho_M \quad (\text{negative effective magnetic charge creates } \mathbf{H} \text{ field lines, positive destroys})$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad (\text{math identity: if a vector field has zero divergence, it is the curl of a vector field})$$

$$\mathbf{B} = \mu \mathbf{H} \quad (\text{linear magnetic material have the applied and total fields linked by a constant})$$

### 3. Derived Equations: Potentials



### 5. Derived Equations: Boundary Conditions

$\nabla \cdot \mathbf{B} = 0$   
 $(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0$

$\nabla \cdot \mathbf{H} = \rho_M$   
 $(\mathbf{H}_2 - \mathbf{H}_1) \cdot \mathbf{n} = \sigma_M$

$\nabla \cdot \mathbf{M} = -\rho_M$   
 $(\mathbf{M}_2 - \mathbf{M}_1) \cdot \mathbf{n} = -\sigma_M$

(using Gaussian pillbox surface)

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_{\text{total}}$   
 $\mathbf{n} \times (\mathbf{B}_2 - \mathbf{B}_1) = \mu_0 \mathbf{K}_{\text{total}}$

$\nabla \times \mathbf{H} = \mathbf{J}$   
 $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}$

$\nabla \times \mathbf{M} = \mathbf{J}_M$   
 $\mathbf{n} \times (\mathbf{M}_2 - \mathbf{M}_1) = \mathbf{K}_M$

(using Amperian loop)

#### 4. Derived Equations: Linear Materials

$$\begin{array}{ccc} \boxed{\mathbf{B} = \mu \mathbf{H}} & & \boxed{\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}} \\ \swarrow \quad \searrow & \blacktriangle \blacktriangle & \swarrow \quad \searrow \\ \boxed{\mathbf{B} = \begin{pmatrix} \mu \mu_0 \\ \mu - \mu_0 \end{pmatrix} \mathbf{M}} & & \boxed{\mathbf{H} = \begin{pmatrix} \mu_0 \\ \mu - \mu_0 \end{pmatrix} \mathbf{M}} \end{array}$$

$$\begin{array}{ccc} \boxed{\mathbf{B} = \mu \mathbf{H}} & & \boxed{\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_{\text{total}}} \\ \swarrow \quad \searrow & \blacktriangle \blacktriangle & \swarrow \quad \searrow \\ \boxed{\nabla^2 \mathbf{A} = -\mu \mathbf{J}} & & \end{array}$$

$$\begin{array}{ccc} \boxed{\mathbf{B} = \mu \mathbf{H}} & & \boxed{(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0} \\ \swarrow \quad \searrow & \blacktriangle \blacktriangle & \swarrow \quad \searrow \\ \boxed{(\mu_2 \mathbf{H}_2 - \mu_1 \mathbf{H}_1) \cdot \mathbf{n} = 0} & & \end{array}$$

$$\begin{array}{ccc} \boxed{\mathbf{B} = \mu \mathbf{H}} & & \boxed{\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}} \\ \swarrow \quad \searrow & \blacktriangle \blacktriangle & \swarrow \quad \searrow \\ \boxed{\mathbf{n} \times \left( \frac{1}{\mu_2} \mathbf{B}_2 - \frac{1}{\mu_1} \mathbf{B}_1 \right) = \mathbf{K}} & & \end{array}$$

In regions of linear *uniform* material and  $\mathbf{J} = 0$ , we can show  $\mathbf{J}_M = 0$ , and therefore  $\mathbf{J}_{\text{total}} = 0$ .

In free space (no magnetic materials),  $\mathbf{M} = 0$ ,  $\mathbf{J}_M = 0$ , and therefore  $\mathbf{B} = \mu_0 \mathbf{H}$  and  $\mathbf{J}_{\text{total}} = \mathbf{J}$ .