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## 1. Scattering Review

- At long wavelengths, we can assume that dipole scattering dominates and that the incident wave, the induced dipoles, and the scattered wave all oscillate in step. In addition, we are typically concerned only with the far-field waves. The effects can be approximated to be instantaneous so that the induced dipoles are the same as is found in electrostatics and magnetostatics.
- The differential scattering cross section with these approximations in place was found to be:

$$\boxed{\frac{d\sigma}{d\Omega}=\frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\boldsymbol{\epsilon}^*\cdot\mathbf{p}+\frac{1}{c}(\hat{\mathbf{k}}\times\boldsymbol{\epsilon}^*)\cdot\mathbf{m}\right|^2}$$

where $\mathbf{p}$ is the electric dipole moment, $\mathbf{m}$ is the magnetic dipole moment, and $\boldsymbol{\epsilon}$ is the unit vector of the polarization of the scattered wave that is being measured.
- The extent to which the scattered wave is polarized is called the average polarization and is defined by:

$$\Pi(\theta)=\frac{\dfrac{d\sigma_{\text{unpol, H}}}{d\Omega}-\dfrac{d\sigma_{\text{unpol, V}}}{d\Omega}}{\dfrac{d\sigma_{\text{unpol, unpol}}}{d\Omega}}$$

## 2. Scattering by a Small Perfectly Conducting Sphere

- Consider a perfectly conducting sphere of radius $a$. If $a$ is much smaller than the wavelength of the incident light, we can use the long-wavelength approximation.
- The electric dipole induced by a uniform incident field was found previously to be:

$$\mathbf{p}=4\pi\epsilon_0 a^3\mathbf{E}_{\text{inc}}(0)$$

- The magnetic moment of a conducting sphere in a uniform magnetic field is:

$$\mathbf{m}=-2\pi a^3\mathbf{H}_{\text{inc}}(0)$$

- Inserting these dipoles into the cross section equation we find:

$$\frac{d\sigma}{d\Omega}=\frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\boldsymbol{\epsilon}^*\cdot\left(4\pi\epsilon_0 a^3\mathbf{E}_{\text{inc}}(0)\right)+\frac{1}{c}(\hat{\mathbf{k}}\times\boldsymbol{\epsilon}^*)\cdot\left(-2\pi a^3\mathbf{H}_{\text{inc}}(0)\right)\right|^2$$

$$\frac{d\sigma}{d\Omega}=\frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\boldsymbol{\epsilon}^*\cdot\left(4\pi\epsilon_0 a^3\boldsymbol{\epsilon}_0 E_0\right)+\frac{1}{c}(\hat{\mathbf{k}}\times\boldsymbol{\epsilon}^*)\cdot\left(-2\pi a^3\frac{1}{\mu_0}\frac{k}{\omega}\hat{\mathbf{k}}_0\times\boldsymbol{\epsilon}_0 E_0\right)\right|^2$$

$$\boxed{\frac{d\sigma}{d\Omega}=k^4 a^6\left|\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0-\frac{1}{2}(\hat{\mathbf{k}}\times\boldsymbol{\epsilon}^*)\cdot\left(\hat{\mathbf{k}}_0\times\boldsymbol{\epsilon}_0\right)\right|^2}$$

- Let us expand this explicitly into the different polarization cases:

$$\frac{d\sigma_{\mathrm{HH}}}{d\Omega}=k^4 a^6\left|1-\frac{1}{2}\cos\theta\right|^2$$

$$\frac{d\sigma_{\mathrm{HV}}}{d\Omega}=0$$

$$\frac{d\sigma_{\mathrm{VH}}}{d\Omega}=0$$

$$\frac{d\sigma_{\mathrm{VV}}}{d\Omega}=k^4 a^6\left|\cos\theta-\frac{1}{2}\right|^2$$

- If unpolarized light is incident, we have:

$$\frac{d\sigma_{\text{unpol, H}}}{d\Omega}=\frac{k^4 a^6}{2}\left|1-\frac{1}{2}\cos\theta\right|^2$$

$$\frac{d\sigma_{\text{unpol, V}}}{d\Omega}=\frac{k^4 a^6}{2}\left|\cos\theta-\frac{1}{2}\right|^2$$

- If unpolarized light is incident and all scattered polarizations are measured, we have:

$$\frac{d\sigma_{\text{unpol, unpol}}}{d\Omega}=\frac{k^4 a^6}{2}\left[\left|1-\frac{1}{2}\cos\theta\right|^2+\left|\cos\theta-\frac{1}{2}\right|^2\right]$$

$$\boxed{\frac{d\sigma_{\text{unpol, unpol}}}{d\Omega}=k^4 a^6\left[\frac{5}{8}(1+\cos^2\theta)-\cos\theta\right]}$$

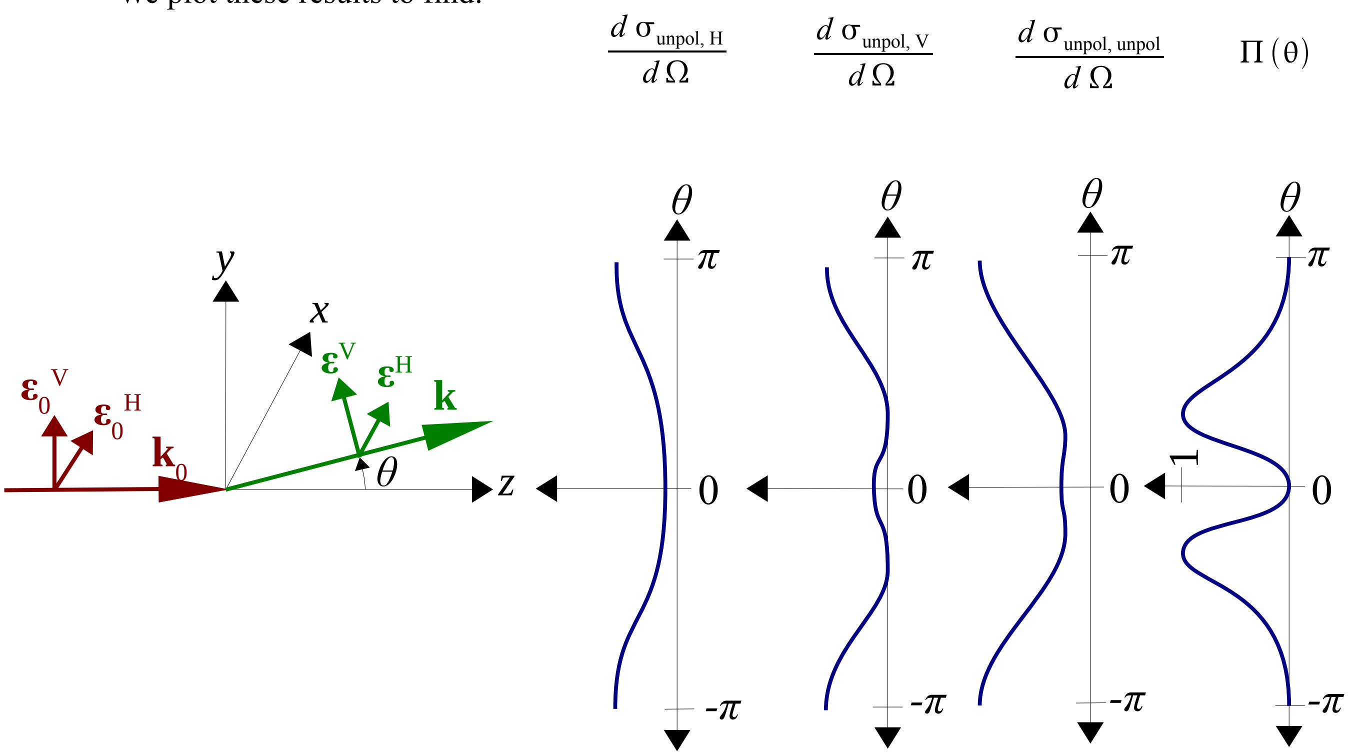
- The average polarization of the scattered radiation is then:

$$\Pi(\theta)=\frac{\dfrac{d\sigma_{\text{unpol, H}}}{d\Omega}-\dfrac{d\sigma_{\text{unpol, V}}}{d\Omega}}{\dfrac{d\sigma_{\text{unpol, unpol}}}{d\Omega}}$$

$$\Pi(\theta)=\frac{\frac{k^4a^6}{2}\left|1-\frac{1}{2}\cos\theta\right|^2-\frac{k^4a^6}{2}\left|\cos\theta-\frac{1}{2}\right|^2}{k^4a^6\left[\frac{5}{8}(1+\cos^2\theta)-\cos\theta\right]}$$

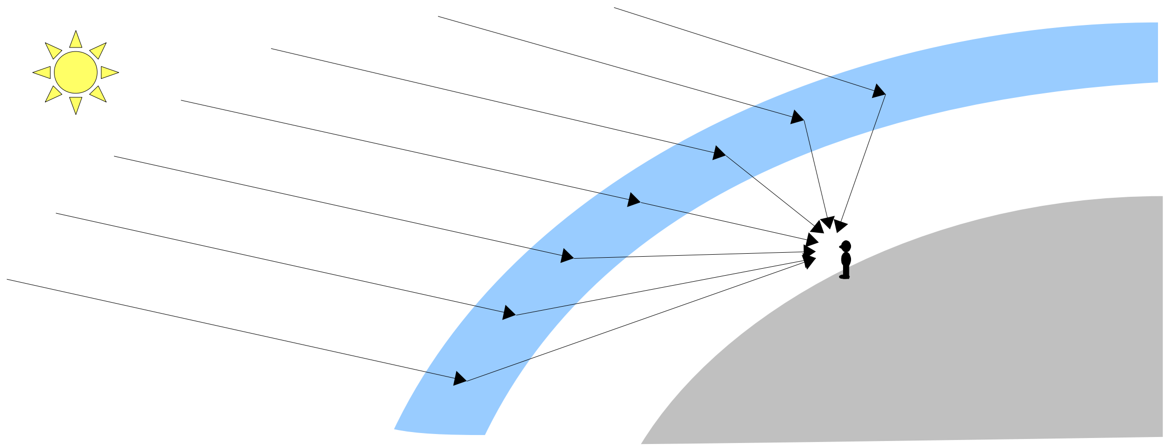
$$\boxed{\Pi(\theta)=\frac{3\sin^2\theta}{5(1+\cos^2\theta)-8\cos\theta}}$$

- We plot these results to find:



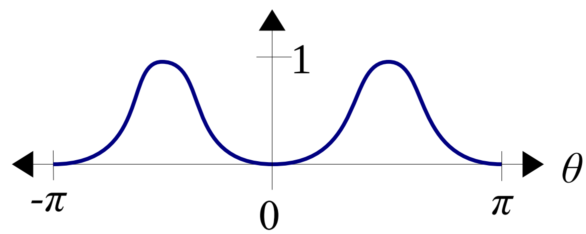

## 3. Atmospheric Scattering Introduction

- The earth's atmosphere is composed of gas molecules, 99% of which are nitrogen ($N_2$) and oxygen ($O_2$) molecules.
- To a good approximation, each molecule scatters sunlight like a small dielectric sphere. We can thus understand atmospheric scattering by using all of the results we found for the scattering due to a small dielectric sphere.
- As the simplest model, we keep only the dominant first bounces. The power of waves that have scattered multiple times are weak enough that they can be neglected. Also assume that the sun is so far away that its light hits the earth as a beam of parallel rays.

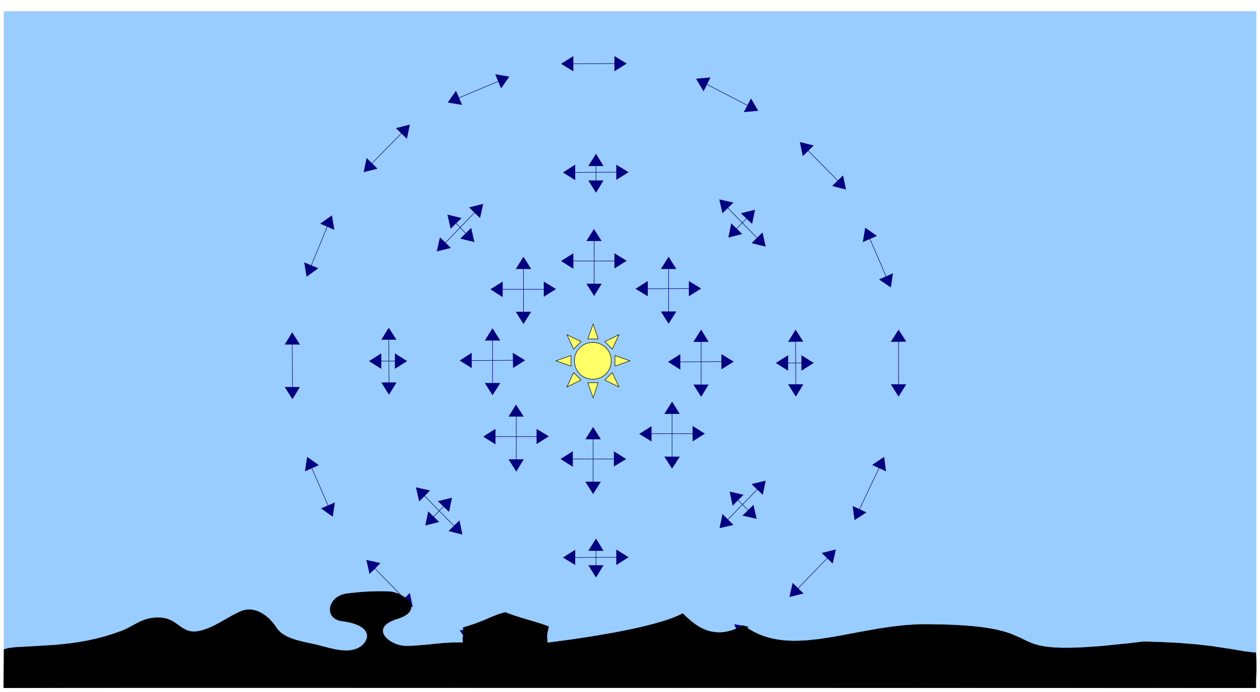


- The incident sunlight is unpolarized but becomes polarized by the dielectric spheres (gas molecules) in the atmosphere. We already found the average polarization of the light scattered by a small dielectric sphere to be:

$$\Pi(\theta)=\frac{\sin^2\theta}{1+\cos^2\theta}$$

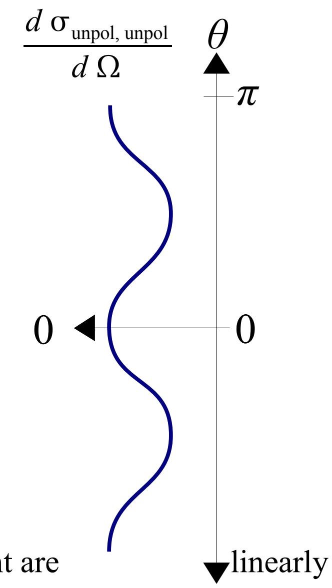



- Looking at the illustration above it becomes obvious that the scattering angle equals the angle between the observer's viewing direction and the direction of the sun.
- We can therefore conclude that the regions of the sky that are closer to the sun are less polarized and that regions closer to a 90 degree viewing angle are more polarized, with 100% polarization occurring when we are looking at the sky in a direction that is at a right angle from the sun.
- This is what you would see looking up at the sky if you could see polarization:



- At noon at the equator, the sun is directly overhead and the sky along the horizon is perpendicular to the sun's direction, so that the light coming from the sky along the horizon is polarized parallel to the horizon.
- At sunset, the sun is on the western horizon and the perpendicular direction is straight up. Looking straight up at the sky we see light that is polarized along the north-to-south line.
- At sunset, if we look at the sky due east (looking completely away from the sun), this corresponds to a scattering angle of 180°, so that we would expect to see completely unpolarized light again. In reality, this light is only mostly unpolarized. The reason for the complication is that the sunlight from such a backward direction must pass through the atmosphere twice.
- Honeybees can see the polarization of the sky and use it to communicate to other bees the direction of food.
- Let us look at the total power radiated into all polarizations. We already found for the small dielectric sphere that:

$$\frac{d\sigma_{\text{unpol, unpol}}}{d\Omega} = k^4 a^6 \left(\frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0}\right)^2 \frac{1}{2}(1+\cos^2\theta)$$




- This means that the parts of the sky closer to the sun (and the corresponding opposite points in the sky) are brighter. The sky directly near the sun should be twice as bright as the sky we see when looking at a right angle to the sun.
- The power scattered in Rayleigh scattering was shown to be proportional to the fourth power of the wave number:

$$\sigma \propto k^4$$

- Because in free space the wave number and the frequency of the light are trivially related, this is equivalent to saying:

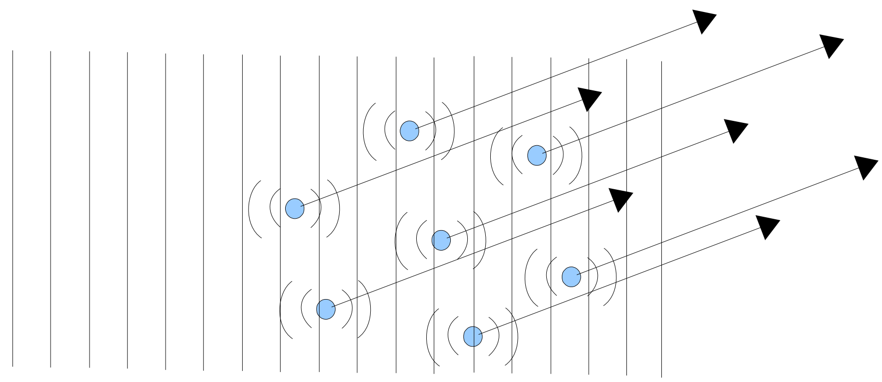
$$\sigma \propto \omega^4$$

- Higher-frequency components of sunlight, such as blue and violet, are scattered more strongly than lower-frequency components such as red.
- Sunlight incident on the atmosphere is white, which is a mixture of all colors. The sky then scatters the blue and violet waves the most and the sky appears whitish blue. The sky does not appear violet because human eyes have low sensitivity to violet.
- If the light traverses enough atmosphere, almost all of the blue waves get scattered out of the beam and the only light left to scatter is lower in frequency, such as red. At sunset, the sun is low and the light reaching us must pass through more atmosphere. That is why the sun and sky appear red at sunset.

## 4. Atmospheric Bulk Attenuation

- First note that we will neglect the contribution to the scattered signal from every wave that has scattered off of multiple scatterers. We are therefore only including single-bounce scattered waves. This is a good approximation because every multi-bounce scattered waves are very weak and rare when the scatterers are spread far apart.
- Every scattering event scatters the forward-traveling wave in all directions. The scattered

wave in a given direction is therefore much weaker than the original wave. A wave that has been scattered off of two scatters in a row has been weakened twice and is therefore much weaker than the single-bounce scattered waves. For this reason, we ignore multi-bounce waves.
- Bulk attenuation means that as an electromagnetic wave traverses past several scatterers, the forward-traveling wave is weakened because the scatterers deflect energy away from the forward direction.



-The scatterers are so numerous that a plot of the forward-traveling wave's intensity versus distance traveled will be a smooth curve:

$$I_{\text{unscat}} = I_0 e^{-\alpha x}$$

- The total scattered wave in a certain direction is going to be the vector sum of the electric fields of all the individual scattered waves.
- If the scatterers are arranged in a certain way, their scattered waves will have different phases, and the total wave can therefore exhibit an interference pattern.
- In terms of the scattered power, the interference effects are included by squaring the sum of the phase differences, which is called the structure factor *F*:

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{system}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{single}} F(\mathbf{q}) \text{ where } F(\mathbf{q}) = \left|\sum_j e^{i\mathbf{q}\cdot\mathbf{x}_j}\right|^2$$

- Here, **q** is the difference vector between incident and scattered wave-vectors.
- If the scatterers are distributed randomly, however, the cross-terms in the structure factor go away, so that the structure factor reduces down to the total number of scatterers *N*, leading to:

$$\sigma_{\text{total}} = \sigma N$$

- Such a case is said to be an incoherent system, or a system with no wave interference effects. As a rule of thumb, we must vector sum electric fields for coherent systems, but must only scalar sum the wave powers for incoherent systems because the cross-terms go away.
- The attenuation coefficient of bulk attenuation is the total power removed from the forward beam per unit volume:

$$\alpha = \frac{\sigma_{\text{total}}}{V} = \frac{\sigma N}{V} = \sigma N_V \quad \text{where } N_V \text{ is total number of molecules per unit volume } V$$

so that the bulk attenuation becomes:

$$I_{\text{unscat}} = I_0 e^{-\sigma N_V x}$$

- The total and differential scattering cross sections of a single dielectric molecule that is part of a collection can be shown to be:

$$\sigma = \frac{2k^4}{3\pi N_V^2}|n-1|^2 \quad \text{and} \quad \frac{d\sigma}{d\Omega} = \frac{k^4}{4\pi^2 N_V^2}|n-1|^2 \frac{1}{2}(1+\cos^2\theta)$$

- These equations are essentially the equations that we have already derived (i.e. the cross section of a small dielectric sphere), but the constants out front have been changed so that we can use the effective index of refraction *n* of the entire dielectric-spheres-in-free-space system rather than the dielectric constant of the sphere material alone.
- Note that these equations may be a little misleading. Taken at face value, they seem to say that more molecules per unit volume $N_V$ will lead to less scattering, which doesn't make sense.
- The key is that the effective index of refraction *n* of the whole system depends on the number of molecules.
- To a first order approximation, the effective index of refraction of air at any altitude is found empirically to equal the molecular density at that altitude times a constant:

$$n-1 = N_V \left( \frac{n_{\text{sea level}} - 1}{N_{V,\text{sea level}}} \right)$$

- Plugging this in we find:

$$\sigma = \frac{2k^4}{3\pi} \left( \frac{n_{\text{sea level}} - 1}{N_{V,\text{sea level}}} \right)^2 \quad \text{and} \quad \frac{d\sigma}{d\Omega} = \frac{k^4}{4\pi^2} \left( \frac{n_{\text{sea level}} - 1}{N_{V,\text{sea level}}} \right)^2 \frac{1}{2}(1+\cos^2\theta)$$

which leads to:

$$I_{\text{unscat}} = I_0 e^{-\alpha x} \quad \text{where} \quad \alpha = N_V \frac{2k^4}{3\pi} \left( \frac{n_{\text{sea level}} - 1}{N_{V,\text{sea level}}} \right)^2$$

- The attenuation equation as written above only holds if the attenuation factor $\alpha$ is constant throughout the entire atmosphere, which would require the atmospheric density $N_v$ to be constant, which it isn't.
- In the earth's atmosphere, the air's density changes as a function of altitude, so that the attenuation factor changes as well.
- The way to handle this is to treat the full width of the atmosphere as a series of slices, each with a constant density and therefore constant attenuation factor, and then let the slice width approach zero. The reduced-power forward wave that exits one slice becomes the incident wave for the next slice, leading to:

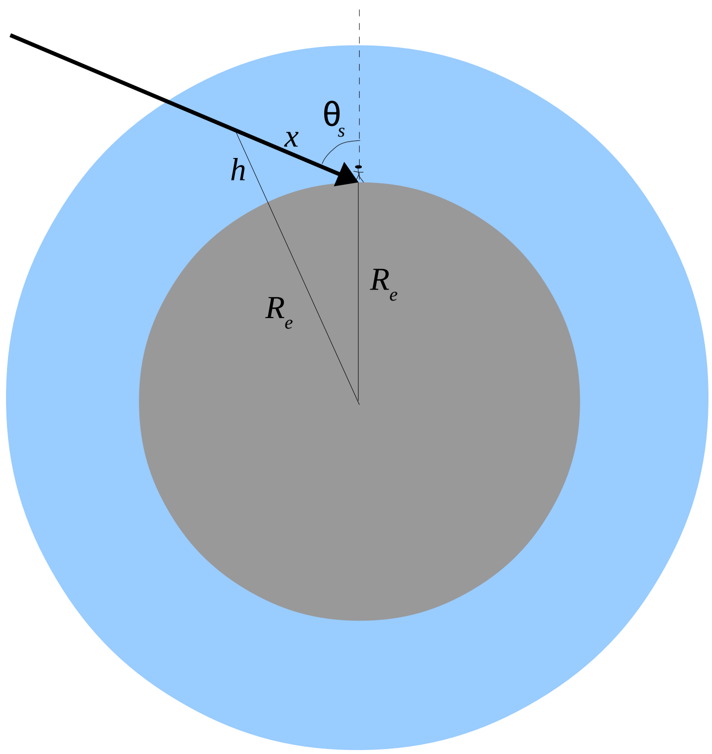
$$I_{\text{unscat}} = I_0 e^{-\alpha(x_0)\Delta x} e^{-\alpha(x_1)\Delta x} e^{-\alpha(x_2)\Delta x} \ldots$$

$$I_{\text{unscat}} = I_0 \exp\left(-\sum_i \alpha(x_i)\Delta x\right)$$

$$I_{\text{unscat}} = I_0 \exp\left(-\int \alpha(x)\, dx\right)$$

$$I_{\text{unscat}} = I_0 \exp\left(\frac{-2k^4}{3\pi}\left(\frac{n_{\text{sea level}}-1}{N_{V,\text{ sea level}}}\right)^2 \int N_V(x)\, dx\right)$$

- If a wave travels at an angle through the atmosphere (not straight down), then *dx* is the actual oblique distance traveled, not the altitude change. We would like to relate it to the altitude change, which requires some geometry.




- In this diagram, *h* is the altitude, $R_e$ is the radius of the earth, and *x* is the actual distance traveled by a ray at this angle going from some point to the ground. Here, $\theta_s$ is the polar angle that the sun makes from the observer's vertical direction, not to be confused with the scattering angle.
- Applying the law of cosines to the triangle shown:

$$(h+R_e)^2 = R_e^2 + x^2 - 2R_e x\cos(\pi - \theta_s)$$

$$x^2 + (2R_e\cos\theta_s)x + (-h(h+2R_e)) = 0$$

$$\boxed{x = -R_e\cos\theta + \sqrt{R_e^2\cos^2\theta_s + h(h+2R_e)}}$$

- This equation tells us that a wave that makes an angle $\theta_s$ from the vertical and travels from an altitude $h$ to the ground has traveled a distance $x$.
- Note that this model neglects the refraction of the light that occurs as the beam travels through air with different indices of refraction. This approximation is valid because studies show that light rays traversing the atmosphere only bend 0.05 to 0.5 degrees due to refraction.
- Now we take the derivative with respect to $h$ to get an incremental distance for use in the integral:

$$d\,x = \frac{h+R_e}{\sqrt{R_e^2 \cos^2\theta_s + h(h+2R_e)}}\,d\,h$$

- In sun observations, we usually specify the sun's position as an elevation angle $\theta_{el}$ relative to the horizon. Let us make the substitution: $\theta_s = 90° - \theta_{el}$ leading to:

$$d\,x = \frac{h+R_e}{\sqrt{R_e^2 \sin^2\theta_{el} + h(h+2R_e)}}\,d\,h$$

- By using this relation, our attenuation equation now contains an integral over altitude instead of an integral over distance traveled. If we want to calculate the total attenuation through the entire atmosphere, the limits on our integral are zero and infinity.

$$I_{\text{unscat}} = I_0 \exp\left(\frac{-2k^4}{3\pi}\left(\frac{n_{\text{sea level}}-1}{N_{V,\text{sea level}}}\right)^2 \int_0^\infty N_V(h)\frac{h+R_e}{\sqrt{R_e^2 \sin^2\theta_{el} + h(h+2R_e)}}\,d\,h\right)$$

- But what is the density of air on earth $N_V(h)$ as a function of altitude?
- Because of gravity and statistics, to a very good approximation, the density of air is exponential:

$$N_V(h) = N_{V,\text{sea level}}\, e^{-h/h_{\text{eff}}}$$ where $h_{\text{eff}}$ = 8700 m gives good agreement with measurements

- The empirical parameter $h_{\text{eff}}$ is the effective height of the atmosphere if the density were constant. Plugging this in, we have:

$$I_{\text{unscat}} = I_0 \exp\left(\frac{-2k^4 h_{\text{eff}}}{3\pi}\frac{(n_{\text{sea level}}-1)^2}{N_{V,\text{sea level}}}\int_0^\infty \frac{(h+R_e)\,e^{-h/h_{\text{eff}}}}{h_{\text{eff}}\sqrt{R_e^2 \sin^2\theta_{el} + h(h+2R_e)}}\,d\,h\right)$$

- This equation represents the intensity of the unscattered light after it has passed through the entire atmosphere.
- The integral can now in principle be solved. However, it is too complicated to be solved analytically and mist be solved numerically.
- The equation above only contains a bunch of constants, the sun's elevation angle, and the wavenumber. Therefore, upon solving the integral and inserting the numerical values of all the constants, we should have an expression that tells us the forward-traveling intensity after traveling through the entire atmosphere as a function of the sun's elevation angle and the wavenumber.

**5. The Sky Spectrum**

- If I stand on the earth's surface and look at the sky with my eyes or a detector, what is the spectrum of the light that I receive?
- To answer this question we develop a composite model: sunlight with a thermal spectrum is incident on the atmosphere, the sunlight travels through the entire atmosphere and is bulk attenuated the entire way, and then the remaining light is scattered by the last bit of atmosphere into my eyes.
- The differential scattering cross section of a single scatterer was defined by:

$$\frac{d\sigma}{d\Omega} = \frac{r^2 I_{\text{scat,single}}}{I_{\text{incident}}}$$

- Solving this equation for the scattered intensity, we have:

$$I_{\text{scat,single}} = \frac{1}{r^2}\frac{d\sigma}{d\Omega} I_{\text{incident}}$$

- Now sum over N scattered in the last layer of atmosphere to get the total intensity:

$$I_{\text{scat,total}} = \frac{N}{r^2}\frac{d\sigma}{d\Omega} I_{\text{incident}}$$

- For a single dielectric sphere in terms of the effective system index of refraction, we found the differential cross section to be:

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{4\pi^2}\left(\frac{n_{\text{sea level}}-1}{N_{V,\text{sea level}}}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

- Inserting this into the previous equation, we find:

$$I_{\text{scat,total}} = \frac{N}{r^2}\left[\frac{k^4}{4\pi^2}\left(\frac{n_{\text{sea level}}-1}{N_{V,\text{sea level}}}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)\right] I_{\text{incident}}$$

- The intensity that is incident on this final layer of scatterers is the sunlight that has been bulk attenuated all through the atmosphere. We thus plug in the bulk attenuation expression from the previous section as the incident intensity of the above equation to find:

$$I_{\text{scat,total}} = \frac{N}{r^2}\frac{k^4}{8\pi^2}\left(\frac{n_{\text{sea level}}-1}{N_{V,\text{sea level}}}\right)^2 \left(1+\cos^2\theta\right) I_0$$
$$\times \exp\left(\frac{-2k^4 h_{\text{eff}}}{3\pi}\frac{\left(n_{\text{sea level}}-1\right)^2}{N_{V,\text{sea level}}}\int_0^\infty \frac{\left(h+R_e\right)e^{-h/h_{\text{eff}}}}{h_{\text{eff}}\sqrt{R_e^2\sin^2\theta_{el}+h\left(h+2R_e\right)}}\,dh\right)$$

- We care only about the spectral pattern, and not the overall brightness, so we can group all the overall constants out front into one constant $A$ and ignore them.

- Also use $k^4 = \left(\frac{2\pi\nu}{c}\right)^4 n^4 \approx \left(\frac{2\pi\nu}{c}\right)^4 (1.0002)^4 \approx \left(\frac{2\pi\nu}{c}\right)^4$ to find:

$$I_{\text{scat,total}} = A\nu^4 I_0 \exp\left(\frac{-32\pi^3 h_{\text{eff}}\nu^4}{3c^4}\frac{(n_{\text{sea level}}-1)^2}{N_{V,\text{sea level}}}\int_0^\infty \frac{(h+R_e)e^{-h/h_{\text{eff}}}}{h_{\text{eff}}\sqrt{R_e^2\sin^2\theta_{el}+h(h+2R_e)}}dh\right)$$

- Lastly, the sunlight incident on the top of the atmosphere can be modeled as a thermal spectrum from a perfect blackbody radiating at temperature $T_s$ = 5777 K:

$$I_0 \propto \frac{\nu^3}{\exp(h\nu/kT_s)-1}$$

Inserting this expression for a thermal spectrum in place of $I_0$ and again absorbing all the overall constants into the arbitrary constant $A$, we finally end up at:

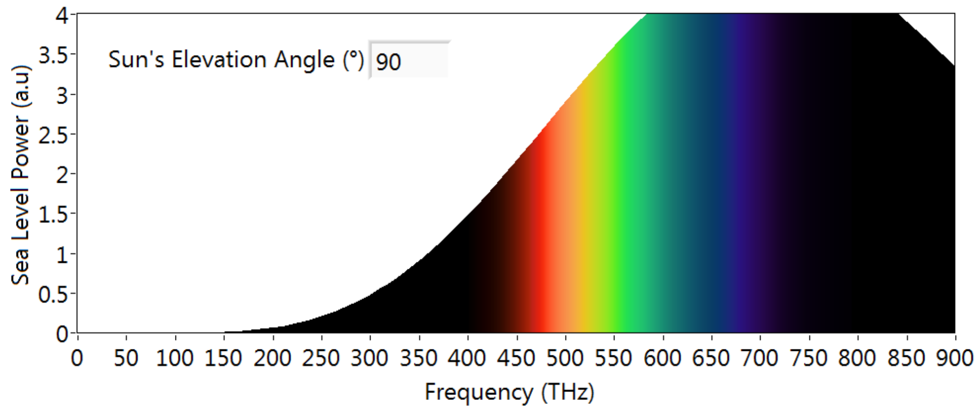
$$\boxed{I_{\text{scat,total}} = A\frac{\nu^7}{\exp(h\nu/kT_s)-1}e^{-\left(\frac{\nu}{\nu_0}\right)^4 L(\theta_{el})}}$$

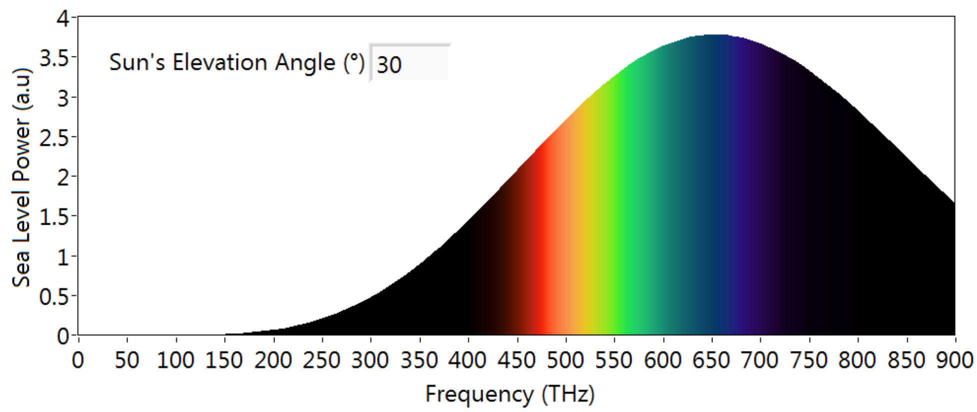
where $\boxed{L(\theta_{el}) = \int_0^\infty \frac{(h+R_e)e^{-h/h_{\text{eff}}}}{h_{\text{eff}}\sqrt{R_e^2\sin^2\theta_{el}+h(h+2R_e)}}dh}$ which is unitless and of order 1

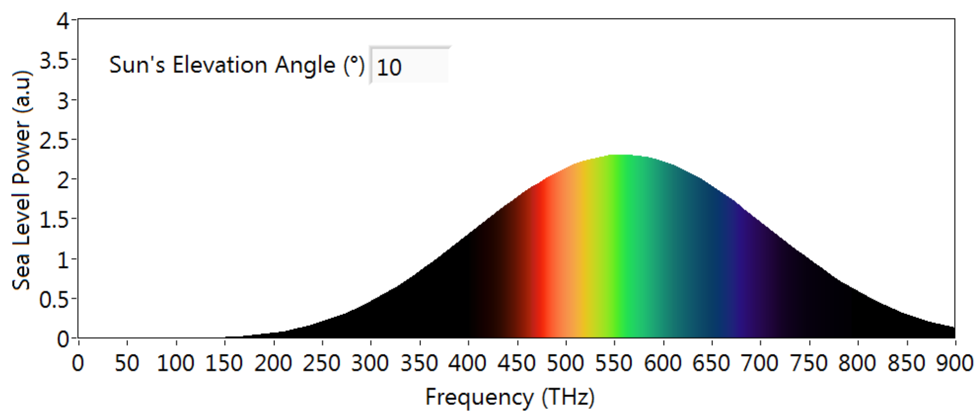
and $\boxed{\nu_0 = \left[\frac{32\pi^3 h_{eff}}{3c^4}\frac{(n_{\text{sea level}}-1)^2}{N_{V,\text{sea level}}}\right]^{-1/4} \approx 950\,\text{THz}}$

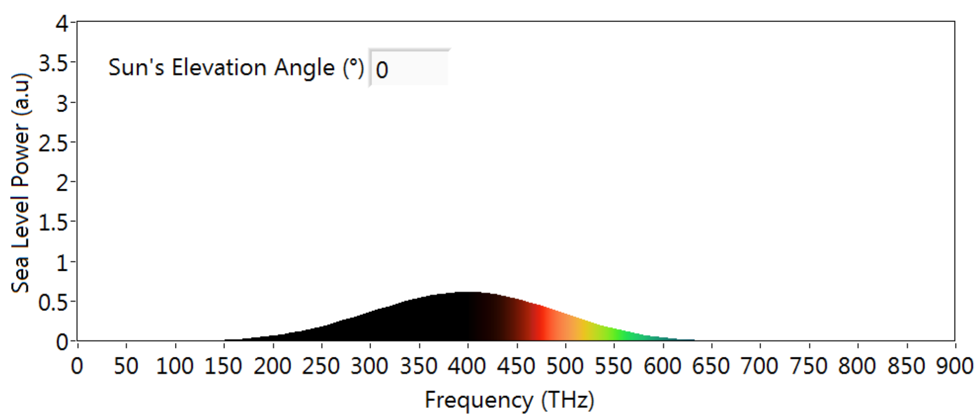
$T_s$ = 5777 K
$n_{\text{sea level}}$ = 1.00030
$N_{\nu,\text{ sea level}}$ = 2.63 x $10^{25}$ molecules/m$^3$,
$R_e$ = 6.371 x $10^6$ m
$h_{\text{eff}}$= 8700 m

- In summary, this equation takes into account the following models and assumptions:
    1. The sunlight hitting the top of the atmosphere is that of a blackbody at $T_s$ = 5777 K.
    2. The density of the air in the atmosphere is exponential in altitude.
    3. The sunlight is not bent due to refraction as is passes through the atmosphere.
    4. Multiple-bounce scattering is ignored.
    5. The sunlight passes through the entire atmosphere and is bulk attenuated along the entire way because of Rayleigh scattering.
    6. At the bottom of the atmosphere, the forward-traveling attenuated sunlight experiences the Rayleigh scattering events that final send light to the observer.
- We plot this equation to see what it predicts for the sky spectrum at different elevations of the sun.


Sun's Elevation Angle (°) 90
Sea Level Power (a.u)
0 0.5 1 1.5 2 2.5 3 3.5 4
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900
Frequency (THz)


Sun's Elevation Angle (°) 30
Sea Level Power (a.u)
0 0.5 1 1.5 2 2.5 3 3.5 4
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900
Frequency (THz)


Sun's Elevation Angle (°) 10
Sea Level Power (a.u)
0 0.5 1 1.5 2 2.5 3 3.5 4
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900
Frequency (THz)


Sun's Elevation Angle (°) 0
Sea Level Power (a.u)
0 0.5 1 1.5 2 2.5 3 3.5 4
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900
Frequency (THz)

- Note that the connection between the physical spectrum of a light beam and what humans experience as “color” is very complicated, so we can only make very general comments.
- As the plots show, between elevation angles of 30° to 90°, the spectrum hardly changes. There is a mixture of colors with blue and violet dominating. We therefore observe the sky during most of the day as whitish blue.
- Considering the spectrum, the day sky should appear whitish violet. It does not because of the way humans experience color: our eyes have poor sensitivity to violet.
- Near 10° the spectrum peaks in the green. Why don't we ever see green skies? The answer is that green is not really dominating. There is an approximately equal mixture of all the colors. Also green is right in the middle of visible spectrum. Humans see a mixture of all color centered on green as white. Therefore, as the sun sets, the sky goes from bluish white to white.
- At sunrise/sunset near 0° elevation, the higher frequency components of the sunlight have all scattered away through bulk attenuation before reaching the scattering event that sends the light to the observer, so that there is only dim red and orange colors left.