Lecture 10 Notes, Electromagnetic Theory II

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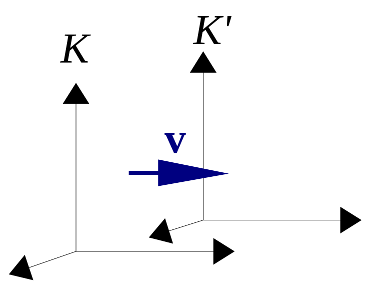
1. Pre-Einstein Relativity

- Einstein did not invent the concept of "relativity," but instead created a more accurate physical theory to describe relativity.
- The concept of relativity describes how the values of physical parameters as measured in one frame of reference relate to the values as measured in another frame of reference.
- The relativity between two frames that have a fixed translation, a fixed rotation, and a fixed scaling is trivial and will not be addressed here. Furthermore, the relativity that involves accelerating frames requires the introduction of inertial forces that complicate the picture, so we will not look at that type of relativity here. Instead, we look at the case of one frame moving at a constant linear velocity relative to another frame. These are called "inertial frames".
- For the purpose of illustration, let us say one frame of reference is always at rest and call it the "lab frame" or "ground frame" K while the other frame of reference is moving and we call it the "moving frame" K'. Frame K' moves at a velocity $\mathbf{v}$ relative to the lab frame. Frame K has coordinates (x, y, z, t) and frame K' has coordinates (x', y', z', t').



- Relativity was first consistently developed by Galilean-Newtonian mechanics.
- Newton's second law ($\mathbf{F} = m\mathbf{a}$) seemed to be valid in all inertial reference frames:

$$F'(\mathbf{x}')=m'\frac{d^2\mathbf{x}'}{dt'^2} \text{ and } F(\mathbf{x})=m\frac{d^2\mathbf{x}}{dt^2}$$

- Galilean-Newtonian mechanics assumed universal force (a force is the same in all inertial reference frames), universal time (all clocks run at the same speed in all inertial reference frames), and universal mass (the mass of an object is the same in all inertial reference frames):

$$F'=F\,,\ t'=t\,,\ m'=m$$

- If the time variables are equal, then incremental time periods in both frames must also be equal. Therefore, the total derivatives with respect to time are also equal:

$$d\,t'=dt\,,\ \frac{d}{d\,t'}=\frac{d}{d\,t}\,,\ \frac{d}{d\,t'^2}=\frac{d}{d\,t^2}\,,\text{ and } \frac{d\,t'}{d\,t}=1$$

- The fact that the total derivative with respect to time is the same in all frames does not imply that the partial derivative with respect to time is the same in all frames. The partial derivative is actually different in different frames as we shall shortly deduce. (A more formal discussion is in the Appendix at the end of this document.)
- Apply these assumptions to Newton's second law:

$$F' = F$$

$$m' \frac{d^2 \mathbf{x}'}{dt'^2} = m \frac{d^2 \mathbf{x}}{dt^2}$$

$$m \frac{d^2 \mathbf{x}'}{dt'^2} = m \frac{d^2 \mathbf{x}}{dt^2}$$

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- This equation leads directly to "Galilean relativity", or the Galilean concept of how to transform parameters from one frame to the next.
- We can simplify the above equation to one dimension and integrate both sides of the equation, being careful to keep integration constants:

$$\frac{dx'}{dt} = \frac{dx}{dt} + A$$

$$x' = x + At + B$$

- The constant B simply tells us where the spatial origin is set when $t = 0$. We can easily make $B = 0$ by aligning the origins of both reference frames at the same point at $t = 0$, leading to:

$$x' = x + At$$

- The constant A must have the dimensions of a velocity, be independent of time and space, and not depend on any particular reference frame. The only thing we know of in this problem that matches these criteria is the velocity v of the moving frame with respect to the lab frame. Setting $A = -v$ (negative because frame K is moving in the negative x direction relative to frame K'), we have:

$\boxed{\mathbf{x}' = \mathbf{x} - \mathbf{v}t}$ and $\boxed{t' = t}$ *Galilean Relativity*

- These equations seem so intuitive that there is a danger of thinking they exist on their own and do not need to be tested.

- We have shown that Galilean relativity is a direct consequence of the following concepts:
    1. The laws of mechanics are the same in all inertial frames
    2. Newton's second law is a correct law of mechanics
    3. Force is universal
    4. Time is universal
    5. Mass is universal
- The principle of Galilean relativity also leads to other statements.
- The length of a moving object as measured in the rest frame is defined as the difference of two endpoints in the rest frame measured at the same time, which we can transform to a new frame:

$$L' = \mathbf{x}_2' - \mathbf{x}_1'$$

$$L' = (\mathbf{x}_2 - \mathbf{v}t) - (\mathbf{x}_1 - \mathbf{v}t)$$

$$L' = \mathbf{x}_2 - \mathbf{x}_1$$

$$\boxed{L' = L}$$

- Galilean relativity therefore leads to the statement that spatial lengths are universal.
- Furthermore, any combination of lengths must therefore be universal in Galilean relativity, such as areas and volumes:

$$A' = A\,,\ \ V' = V$$

- If we take the first time derivative of the Galilean relativity principle, we get:

$$\mathbf{x}' = \mathbf{x} - \mathbf{v}t$$

$$\frac{d\mathbf{x}'}{dt} = \frac{d\mathbf{x}}{dt} - \mathbf{v}$$

$$\frac{d\mathbf{x}'}{dt'} = \frac{d\mathbf{x}}{dt} - \mathbf{v} \quad \text{because} \quad \frac{d}{dt'} = \frac{d}{dt}$$

$$\boxed{\mathbf{u}' = \mathbf{u} - \mathbf{v}} \quad \text{or}$$

$$\boxed{\mathbf{u} = \mathbf{u}' + \mathbf{v}}$$ *Galilean Velocity Addition Formula*

Here **u** is the velocity of an object as measured in frame *K*, **u'** is the velocity of the object as measured in frame *K'*, and **v** is the velocity of frame *K'* relative to frame *K*. The simple linear addition of velocities is one of the most seemingly intuitive parts of pre-Einstein relativity.
- For example, if I ride in a car at 60 mph and throw a baseball forward at 100 mph, according to Galilean relativity, the ball is traveling 160 mph relative to the ground.
- The way that a partial derivative transforms under Galilean relativity can be found by expanding the partial derivative:

$$\frac{\partial}{\partial x'} = \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial x'}\frac{\partial}{\partial y} + \frac{\partial z}{\partial x'}\frac{\partial}{\partial z} + \frac{\partial t}{\partial x'}\frac{\partial}{\partial t}$$

which leads to:

$$\frac{\partial}{\partial x'}=\frac{\partial}{\partial x}$$

- Doing similar expansions for the other spatial coordinates leads to:

$$\frac{\partial}{\partial y'}=\frac{\partial}{\partial y} \text{ and } \frac{\partial}{\partial z'}=\frac{\partial}{\partial z}$$

- Putting these pieces together, we can also show that the gradient and Laplacian operator are the same in all inertial reference frames according to Galilean relativity:

$$\boxed{\nabla'=\nabla} \text{ and } \boxed{\nabla'^2=\nabla^2}$$

- Make an expansion of the partial derivative with respect to time:

$$\frac{\partial}{\partial t'}=\frac{\partial x}{\partial t'}\frac{\partial}{\partial x}+\frac{\partial y}{\partial t'}\frac{\partial}{\partial y}+\frac{\partial z}{\partial t'}\frac{\partial}{\partial z}+\frac{\partial t}{\partial t'}\frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial t'}=v_x\frac{\partial}{\partial x}+v_y\frac{\partial}{\partial y}+v_z\frac{\partial}{\partial z}+\frac{\partial}{\partial t}$$

$$\boxed{\frac{\partial}{\partial t'}=\mathbf{v}\cdot\nabla+\frac{\partial}{\partial t}}$$

This is a very different statement from $\frac{d}{dt'}=\frac{d}{dt}$.

2. Galilean Relativity Transformations for Electric and Magnetic Fields

- When you transform an equation to another inertial frame, *you must transform everything*, including the fields, the sources, and the operators.
- We state up front that Maxwell's equations do not obey Galilean relativity, but rather obey Special Relativity. Therefore, it is not immediately clear how to apply a Galilean transformation to the fields and sources since the whole approach is wrong from the start.
- However, we can get some guidance by requiring Galilean relativity to be the low-speed limit of Special Relativity. Therefore, the Galilean-invariant electromagnetic equations should be the low-speed limit of Maxwell's equations.
- There are two different, valid, low-speed limits to Maxwell's equations: Electro-Quasi-Statics (EQS) and Magneto-Quasi-Statics (EQS). There are therefore two different, valid sets of Galilean transformations for the electromagnetic fields and sources.
- In the EQS formulation, the charge effects dominate over the current effects. Therefore, the charge density is the same in all reference frames. The effect of a frame transformation freezing a current density into a charge density is considered small enough in EQS to be ignored.

$\boxed{\rho'=\rho}$ (EQS Galilean relativity)

- A current density **J** is just a charge density ρ moving at some velocity **u**, **J** = ρ**u**. Therefore, the

EQS Galilean transformation for current density can be found by using the Galilean velocity addition formula:

$$\mathbf{J}'=\rho'\mathbf{u}'$$

$$\mathbf{J}'=\rho\,\mathbf{u}'$$

$$\mathbf{J}'=\rho(\mathbf{u}-\mathbf{v})$$

$$\boxed{\mathbf{J}'=\mathbf{J}-\rho\,\mathbf{v}}\quad\text{(EQS Galilean relativity)}$$

- In the MQS formulation, the current effects dominate over the charge effects. Therefore, the current density is the same in all reference frames. The effect of a frame transformation turning a charge into a current is considered small enough in MQS to be ignored.

$$\boxed{\mathbf{J}'=\mathbf{J}}\quad\text{(MQS Galilean relativity)}$$

- Since currents dominate in MQS, we can't ignore the possibility that a frame transformation will “catch up” with a current density and turn part of it into a charge density, leading to:

$$\boxed{\rho'=\rho-\frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}}\quad\text{(MQS Galilean relativity)}$$

- We have therefore found how the electromagnetic sources transform in the two different Galilean relativity possibilities.
- We now need to find how the fields transform in the two cases. Galilean relativity assumes forces are universal, so that the Lorentz force on the sources must be the same in all frames:

$$\mathbf{F}'=\mathbf{F}$$

$$\int(\rho'\mathbf{E}'+\mathbf{J}'\times\mathbf{B}')\,dV'=\int(\rho\,\mathbf{E}+\mathbf{J}\times\mathbf{B})\,dV$$

- In the EQS framework, the electric field effects are much stronger, so the magnetic field's contribution to the force drops out, leaving:

$$\int\rho'\mathbf{E}'\,dV=\int\rho\,\mathbf{E}\,dV'$$

- If we tried to keep both the electric and magnetic force, then the equations would not obey Galilean relativity.
- Since the charge density and the volume are the same in all frames in the EQS framework, we immediately see that:

$$\boxed{\mathbf{E}'=\mathbf{E}}\quad\text{(EQS Galilean relativity)}$$

- To see how the magnetic field transforms between frames in the EQS Galilean framework, we have to turn to Maxwell's equations. This is valid as long as we stick to the EQS version of Maxwell's equations (i.e. electrostatics plus the no-magnetic-monopole law plus the Ampere-

Maxwell law).
- Start with the Maxwell-Ampere law in the primed frame:

$$\nabla'\times\mathbf{B}'=\frac{1}{c^2}\frac{\partial\mathbf{E}'}{\partial t}+\mu_0\mathbf{J}'$$

- Transform everything possible into the unprimed frame according to the Galilean rules:

$$\nabla\times\mathbf{B}'=\frac{1}{c^2}\left(\mathbf{v}\cdot\nabla+\frac{\partial}{\partial t}\right)\mathbf{E}+\mu_0(\mathbf{J}-\rho\,\mathbf{v})$$

$$\nabla\times\mathbf{B}'=\frac{1}{c^2}(\mathbf{v}\cdot\nabla)\mathbf{E}+\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}+\mu_0\mathbf{J}-\mu_0\rho\,\mathbf{v}$$

$$\nabla\times\mathbf{B}'=\nabla\times\mathbf{B}+\frac{1}{c^2}(\mathbf{v}\cdot\nabla)\mathbf{E}-\mu_0\rho\,\mathbf{v}$$

- Now consider the vector identity: $\nabla\times(\mathbf{a}\times\mathbf{b})=\mathbf{a}(\nabla\cdot\mathbf{b})-\mathbf{b}(\nabla\cdot\mathbf{a})+(\mathbf{b}\cdot\nabla)\mathbf{a}-(\mathbf{a}\cdot\nabla)\mathbf{b}$
- Set **a** = **v** and **b** = **E** in this identity, recognize that the frame velocity **v** is constant so that derivatives of it are zero, and use Gauss's law to replace the divergence of the electric field with the charge density to find:

$$\nabla\times(\mathbf{v}\times\mathbf{E})=\frac{1}{\epsilon_0}\rho\,\mathbf{v}-(\mathbf{v}\cdot\nabla)\mathbf{E}$$

-Multiply this by ($-1/c^2$):

$$-\frac{1}{c^2}\nabla\times(\mathbf{v}\times\mathbf{E})=\frac{1}{c^2}(\mathbf{v}\cdot\nabla)\mathbf{E}-\mu_0\rho\,\mathbf{v}$$

- Insert this identity into the transformed Maxwell-Ampere law to find:

$$\nabla\times\mathbf{B}'=\nabla\times\mathbf{B}-\frac{1}{c^2}\nabla\times(\mathbf{v}\times\mathbf{E})$$

- Integrate away the curl operators everywhere to find:

$$\boxed{\mathbf{B}'=\mathbf{B}-\frac{1}{c^2}\mathbf{v}\times\mathbf{E}}$$ (EQS Galilean relativity)

- Now we need to know how the fields transform in the MQS framework. Return to the statement of universal electromagnetic force:

$$\int(\rho'\mathbf{E}'+\mathbf{J}'\times\mathbf{B}')dV'=\int(\rho\,\mathbf{E}+\mathbf{J}\times\mathbf{B})dV$$

- In MQS, the magnetic field dominates, so we drop the electric field's contribution to the force:

$$\int \mathbf{J}'\times\mathbf{B}'dV' = \int \mathbf{J}\times\mathbf{B}\,dV$$

- Since the current and the volume are the same in all frames in the MQS framework, this immediately tells us that the magnetic field must also be the same in all frames:

$$\boxed{\mathbf{B}'=\mathbf{B}} \quad \text{(MQS Galilean relativity)}$$

- To see how the electric field transforms between frames in the MQS Galilean framework, we have to turn to Maxwell's equations. This is valid as long as we stick to the MQS version of Maxwell's equations (i.e. magnetostatics plus Gauss's law plus Faraday's law).
- Start with Faraday's law:

$$\nabla'\times\mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'}$$

- Use all of our MQS Galilean transformation rules so far to transform to the unprimed frame:

$$\nabla\times\mathbf{E}' = -\left(\mathbf{v}\cdot\nabla + \frac{\partial}{\partial t}\right)\mathbf{B}$$

$$\nabla\times\mathbf{E}' = -(\mathbf{v}\cdot\nabla)\mathbf{B} - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{E}' = \nabla\times\mathbf{E} - (\mathbf{v}\cdot\nabla)\mathbf{B}$$

- Use the same vector identity again: $\nabla\times(\mathbf{a}\times\mathbf{b}) = \mathbf{a}(\nabla\cdot\mathbf{b}) - \mathbf{b}(\nabla\cdot\mathbf{a}) + (\mathbf{b}\cdot\nabla)\mathbf{a} - (\mathbf{a}\cdot\nabla)\mathbf{b}$
- This time set **a** = **v** and **b** = **B**, apply the fact that the derivative of the frame velocity is zero, and use the no-magnetic-monopole law to drop the term with the divergence of **B** to find:

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}\cdot\nabla)\mathbf{B}$$

- Insert this into the transformed Faraday's law to find:

$$\nabla\times\mathbf{E}' = \nabla\times\mathbf{E} + \nabla\times(\mathbf{v}\times\mathbf{B})$$

- Integrate away the curl operators everywhere to find:

$$\boxed{\mathbf{E}'=\mathbf{E}+(\mathbf{v}\times\mathbf{B})} \quad \text{(MQS Galilean relativity)}$$

- In summary, the electromagnetic fields and sources transform under Galilean relativity according to the following rules:

Electro-Quasi-Statics	Magneto-Quasi-Statics
$\rho' = \rho$	$\rho' = \rho - \frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}$
$\mathbf{J}' = \mathbf{J} - \rho\,\mathbf{v}$	$\mathbf{J}' = \mathbf{J}$
$\mathbf{E}' = \mathbf{E}$	$\mathbf{E}' = \mathbf{E} + (\mathbf{v}\times\mathbf{B})$
$\mathbf{B}' = \mathbf{B} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}$	$\mathbf{B}' = \mathbf{B}$

- Note that some textbooks try to lump these two different cases of electromagnetic Galilean relativity into one case and state that the transformation equations $\mathbf{E}' = \mathbf{E} + (\mathbf{v}\times\mathbf{B})$ and $\mathbf{B}' = \mathbf{B} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}$ can be used at the same time. This is incorrect. These two equations do not simultaneously obey the Galilean relativity requirements. Furthermore, these two equations do not even obey a basic requirement of any relativity system: the group law (i.e. two successive frame transformations should be equivalent to a single larger frame transformation). For further information, see M. Le Bellac and J. M. Levy-Leblond, “Galilean Electromagnetism,” Il Nuovo Cimento. *14*, 217-233 (1973).

<u>3. Galilean Relativity Applied to Maxwell's Equations</u>

- Now that we know how the fields, sources, and operators transform under Galilean relativity, we can see whether Maxwell's equations in their full form obey Galilean relativity.
- In the EQS framework, we already know that the Ampere-Maxwell law and Gauss's law obey Galilean relativity since we specifically constructed the transformations to obey these laws.
- We now test whether Faraday's law obeys EQS Galilean relativity:

$$\nabla'\times\mathbf{E}' = -\frac{\partial\mathbf{B}'}{\partial t'}$$

- Apply the EQS Galilean transformation rules to find:

$$\nabla\times\mathbf{E} = -\left(\mathbf{v}\cdot\nabla + \frac{\partial}{\partial t}\right)\left(\mathbf{B} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}\right)$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} - (\mathbf{v}\cdot\nabla)\mathbf{B} + \frac{1}{c^2}(\mathbf{v}\cdot\nabla)\mathbf{v}\times\mathbf{E} + \frac{1}{c^2}\mathbf{v}\times\frac{\partial\mathbf{E}}{\partial t}$$

- This equation does not have the same form as Faraday's law. Therefore, Faraday's law does not obey Galilean relativity in the EQS approach.
- Now test the no-magnetic-monopole law for EQS Galilean relativity:

$$\nabla'\cdot\mathbf{B}' = 0$$

$$\nabla\cdot\left(\mathbf{B} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}\right) = 0$$

$$\nabla\cdot\mathbf{B}=\frac{1}{c^2}\nabla\cdot(\mathbf{v}\times\mathbf{E})$$

Using a vector identity on the right-hand side, we find:

$$\nabla\cdot\mathbf{B}=-\frac{1}{c^2}\mathbf{v}\cdot(\nabla\times\mathbf{E})$$

- Using Faraday's law, we replace the curl of the electric field on the right to find:

$$\nabla\cdot\mathbf{B}=\frac{1}{c^2}\mathbf{v}\cdot\left(\frac{\partial\mathbf{B}}{\partial t}\right)$$

- In order to have the same form as the standard no-magnetic-monpole law, the term on the right would have to be zero. Therefore, the no-magnetic-monpole law also does not obey Galilean relativity in the EQS approach.
- The part that breaks Galilean relativity in both Faraday's law and the no-magnetic-monopole law is the time derivative of **B**. This is exactly the piece that is thrown away to reduce Maxwell's equations to the EQS approximation.
- Therefore, whenever the magnetic field of a system changes so slowly in time that its time derivative can be approximated to be zero, Special Relativity effects go away and Maxwell's equations reduce down to the equations of electro-quasi-statics, which obey Galilean relativity.

- In the MQS framework, we already know that Faraday's law and the no-magnetic-monopole law obey Galilean relativity, since we specifically designed the rules this way.
- We now test Gauss's law for MQS Galilean relativity:

$$\nabla'\cdot\mathbf{E}'=\frac{\rho'}{\epsilon_0}$$

$$\nabla\cdot(\mathbf{E}+(\mathbf{v}\times\mathbf{B}))=\frac{1}{\epsilon_0}\left(\rho-\frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}\right)$$

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}-\frac{1}{\epsilon_0}\frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}-\nabla\cdot(\mathbf{v}\times\mathbf{B})$$

- Use a vector identity on the last term to find:

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}-\frac{1}{\epsilon_0}\frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}+\mathbf{v}\cdot(\nabla\times\mathbf{B})$$

- Use the Maxwell-Ampere law to replace the curl of the magnetic field and find:

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}-\frac{1}{\epsilon_0}\frac{1}{c^2}\mathbf{v}\cdot\mathbf{J}+\mathbf{v}\cdot\left(\mu_0\mathbf{J}+\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}+\frac{1}{c^2}\mathbf{v}\cdot\frac{\partial\mathbf{E}}{\partial t}$$

- In order to have the same form as the standard Gauss's law, the last term would have to go away. Therefore, Gauss's law does not obey Galilean relativity in the MQS approach.
- We now test the Ampere-Maxwell law:

$$\nabla'\times\mathbf{B}'=\mu_0\mathbf{J}'+\frac{1}{c^2}\frac{\partial\mathbf{E}'}{\partial t'}$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}+\frac{1}{c^2}\left(\mathbf{v}\cdot\nabla+\frac{\partial}{\partial t}\right)(\mathbf{E}+(\mathbf{v}\times\mathbf{B}))$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}+\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}+\frac{1}{c^2}(\mathbf{v}\cdot\nabla)\mathbf{E}+\frac{1}{c^2}(\mathbf{v}\cdot\nabla)(\mathbf{v}\times\mathbf{B})+\frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{v}\times\mathbf{B})$$

- This obviously does not have the same form as the Ampere-Maxwell law.
- Therefore, the Ampere-Maxwell law does not obey Galilean relativity in the MQS approach.
- The part that breaks Galilean relativity in both Gauss's law and the Ampere-Maxwell law is the time derivative of **E**. This is exactly the piece that is thrown away to reduce Maxwell's equations to the MQS approximation.
- Therefore, whenever the electric field of a system changes so slowly in time that its time derivative can be approximated to be zero, Special Relativity effects go away and Maxwell's equations reduce down to the equations of magneto-quasi-statics, which obey Galilean relativity.
- In summary, Maxwell's equations do not in general obey Galilean relativity.

- Since Maxwell's equations do not obey Galilean relativity, which was derived from Newton's laws, we have contradicting descriptions of the physical world.
- There are a few possibilities to resolve this contradiction:
    1. Maxwell's equations are wrong. We must fix Maxwell's equations to obey Galilean relativity.
    2. Our assumption of inertial frames in electrodynamics was wrong. There is a preferred reference frame in which the medium that propagates the waves is at rest.
    3. Galilean relativity and Newton's laws are wrong.
- Historically speaking, the first option was hardly plausible because there was so much experimental evidence supporting Maxwell's equations.
- The third option seemed too radical.
- Most physicists turned to option two as the best explanation.
- They reasoned that just like sound is a vibration of air, light must be a vibration of some medium they dubbed the "luminiferous ether".
- Efforts commenced to detect and better understand the ether.

4. The Ether

- The ether had to be defined as a substance that is always at rest in some universal frame.
- All efforts to detect the ether failed.
- Efforts turned to trying to indirectly detect the ether by its effect on light.
- Because the earth orbits in a circle and is always changing its velocity, it cannot always be in

this universal ether rest frame.
- The ether must be moving linearly with respect to any frame of reference on earth.
- Light traveling in the direction of the ether would travel at a different speed than light traveling in a perpendicular direction.
- The Michelson-Morley experiment used an interferometer to split light, send it in perpendicular directions, recombine it and measure any difference in their speeds as an interference pattern.
- This experiment found no evidence of velocity difference and thus strong evidence that there is no ether.
- This result was historically assumed to be caused by experimental errors. Many more experiments were performed along these lines, each with increasing accuracy, and each demonstrating that no ether exists.

5. Einstein's Postulates

- Although the Michelson-Morley experiment and subsequent experiments were disproving the ether concept, Einstein claims he was not motivated by these experiments.
- Einstein instead chose to pursue option three, that Galilean relativity and Newton's laws are wrong. He claimed to have been motivated to find a more harmonious and logical theory that matches the relativity obeyed by Maxwell's equations.
- Replacing Galilean relativity involved removing some of the assumptions inherent in Galilean relativity and using new postulates.
- Einstein used as his first postulate of Special Relativity:

1. The laws of physics (including electrodynamics) are the same in all inertial frames.

- This was a safe postulate because all experiments and theories seemed to support this.
- This postulate immediately ruled out any special universal reference frame such as the ether.
- Einstein discarded the rest of the assumptions of Galilean relativity: that time is universal and that object lengths are universal.
- Doing so, he opened up his principle of relativity to the possibility that clocks can run at different speeds and that the same object can have different lengths depending on which frame it is measured in.
- The challenge before Einstein was now to come up with a second postulate that would lead to a relativity principle that would make Maxwell's equations hold the same in all frames.
- Einstein chose as his second postulate:

2. The speed of light is a constant no matter in what reference frame it is measured.

- This postulate is written in its historical form. However, it ultimately is equivalent to the more general postulate: “There is a universal speed limit”.
- This postulate seems counter-intuitive and its consequences are profound. It took the boldness and genius of Einstein to attempt such a line of thought.
- If I am driving in a car close to the speed of light c, and turn on my headlights, from my perspective the light will still be emitted and travel away from me at c.
- But according to Einstein's relativity, the light is also traveling at speed c relative to the ground. This is only possible if time and space are different in different frames.
- Since the time when Einstein proposed this postulate (1905) there has been considerable experimental evidence that the speed of light is constant in all frames.

APPENDIX

- We should be careful to remember the difference between a partial and full derivative.
- A full derivative operates on all variables in a function:

$$\frac{d\mathbf{x}}{dt}=\frac{dx}{dt}\hat{\mathbf{i}}+\frac{dy}{dt}\hat{\mathbf{j}}+\frac{dz}{dt}\hat{\mathbf{k}} \text{ so that } \mathbf{v}=v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}+v_z\hat{\mathbf{k}}$$

- A partial derivative only operates on the one variable of interest and holds all others constant:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial t}\left(2x+y^3+xt^2\right)=2xt$$

- There is a difference between a variable and a function. Partial derivatives only treat variables as constants and not functions. For instance, in the equation

$$x=x'+v_x t$$

the symbol x' is a variable and the symbol x is a function. Therefore $\frac{\partial x'}{\partial t}$ for the above equation gives zero because x' is a variable, is therefore treated as a constant, and the derivative of a constant is zero. But $\frac{\partial x}{\partial t}$ for the above equation gives v_x because x is a function in this context and cannot be treated as constant. With this in mind, both the full derivative and the partial derivative can be expanded into another basis using the chain rule:

$$\frac{\partial}{\partial t'}=\frac{\partial x}{\partial t'}\frac{\partial}{\partial x}+\frac{\partial y}{\partial t'}\frac{\partial}{\partial y}+\frac{\partial z}{\partial t'}\frac{\partial}{\partial z}+\frac{\partial t}{\partial t'}\frac{\partial}{\partial t} \text{ and}$$

$$\frac{d}{dt'}=\frac{dx}{dt'}\frac{\partial}{\partial x}+\frac{dy}{dt'}\frac{\partial}{\partial y}+\frac{dz}{dt'}\frac{\partial}{\partial z}+\frac{dt}{dt'}\frac{\partial}{\partial t}$$

Notice that the partial derivative expansion and the full derivative expansion are very similar, but the one uses partial derivatives in the chain rule while the other uses total derivatives.
- Let us apply these expansions to Galilean relativity, $\mathbf{x}=\mathbf{x}'+\mathbf{v}t'$, $t'=t$.
- For the partial derivative, terms like $\frac{\partial x}{\partial t'}$ lead to $\frac{\partial x}{\partial t'}=v_x$, so that:

$$\frac{\partial}{\partial t'}=\frac{\partial x}{\partial t'}\frac{\partial}{\partial x}+\frac{\partial y}{\partial t'}\frac{\partial}{\partial y}+\frac{\partial z}{\partial t'}\frac{\partial}{\partial z}+\frac{\partial t}{\partial t'}\frac{\partial}{\partial t} \text{ becomes:}$$

$$\frac{\partial}{\partial t'}=v_x\frac{\partial}{\partial x}+v_y\frac{\partial}{\partial y}+v_z\frac{\partial}{\partial z}+\frac{\partial}{\partial t}$$

$$\boxed{\frac{\partial}{\partial t'}=\mathbf{v}\cdot\nabla+\frac{\partial}{\partial t}}$$

- For the full derivative, terms like $\frac{dx}{dt'}$ lead to $\frac{dx}{dt'}=\frac{dx'}{dt'}+v_x=u_x'+v_x$, so that:

$$\frac{d}{dt'}=\frac{dx}{dt'}\frac{\partial}{\partial x}+\frac{dy}{dt'}\frac{\partial}{\partial y}+\frac{dz}{dt'}\frac{\partial}{\partial z}+\frac{dt}{dt'}\frac{\partial}{\partial t}$$ becomes:

$$\frac{d}{dt'}=(u_x'+v_x)\frac{\partial}{\partial x}+(u_y'+v_y)\frac{\partial}{\partial y}+(u_z'+v_z)\frac{\partial}{\partial z}+\frac{\partial}{\partial t}$$

$$\frac{d}{dt'}=(\mathbf{u}'+\mathbf{v})\cdot\nabla+\frac{\partial}{\partial t}$$

- Now use the velocity addition formula to find:

$$\boxed{\frac{d}{dt'}=\mathbf{u}\cdot\nabla+\frac{\partial}{\partial t}}$$

- Expand the full derivative with respect to *t* in the unprimed basis as well:

$$\frac{d}{dt}=\frac{dx}{dt}\frac{\partial}{\partial x}+\frac{dy}{dt}\frac{\partial}{\partial y}+\frac{dz}{dt}\frac{\partial}{\partial z}+\frac{dt}{dt}\frac{\partial}{\partial t}$$

$$\boxed{\frac{d}{dt}=\mathbf{u}\cdot\nabla+\frac{\partial}{\partial t}}$$

- Comparing the two above equations in boxes, we immediately find:

$$\boxed{\frac{d}{dt'}=\frac{d}{dt}}$$