COMBINING TIME SERIES COMPONENTS FOR IMPROVED FORECASTS.

Frank G. Landram†.
T. Boone Pickens College of Business, West Texas A&M University, Canyon, Texas 79016
email: flandram@mail.wtamu.edu

Robert J. Pavur,
College of Business Administration, University of North Texas, Denton, Texas, 76203
email, pavur@unt.edu

Bahram Alidaee,
The University of Mississippi, School of Business Administration, University, Ms 38677
email: Balidaee@bus.olemiss.edu

INTRODUCTION

Most agree that improved accuracy is obtained by combining forecasts. However, the manner in which these forecasts are best combined is still being explored. This article promotes the combination of time series components along with other forecasts in least squares modeling. Albritton and McMullen (2006) present an excellent article on combining a curvilinear trend with seasonal indices using Microsoft Excel Solver. These authors compare their approach with the traditional time series decomposition approach. The error sum of squares (SSE) for these methods were 266,592 and 306,469, respectively. They conclude their approach with Excel Solver reducing SSE by 13%. In this paper, the same data are used in combining time series components within a least squares equation reducing SSE by 39% to 186,253. This article illustrates the benefits of using least squares models to combine forecasts with time series components. This approach to forecasting is easy to understand, simple to use, and works well in elementary statistics and operations management courses. Another benefit is that these forecasting models are easily derived using the pedagogy inherent in spreadsheet regression.

Originality. The use of time series components in least squares equations is known in many circles through oral tradition and pragmatic intuition. Surprisingly, this productive tool of forecasting does not frequent the literature. Although used decades ago at the Oklahoma Bureau of Business Research, it has become one of forecasting's best kept secrets. However, once it becomes known, this innovative tool of forecasting is destined to become extremely popular in the classroom and in the workplace.

Forecasting. Accurate forecasts are extremely important in business and industry. However, each business must select the forecasting methods that help their particular situation. This forecasting dilemma is further complicated by the fact that most economic conditions are constantly changing. Therefore, the practice of combining forecasts that are conducive to a variety of economic conditions has gained popularity

(Batchelor and Dua, 1995). Unrestricted least squares is an extremely popular and highly regarded method of combining forecasts (Granger and Ramanathan, 1984). This paper describes improvements to this effective forecasting method. The use of a single seasonal index rather than multiple (11 for monthly) indicator variables has only recently been committed to writing (Landram. et al.,2004) and is extended in this article. These improved methods expand the capabilities of combined forecast models enabling them to become more practical and effective.

Consider the quarterly forecasting equation

$$\hat{Y}_t = b_0 + b_1X_t + b_2S_j + b_3C_t \qquad (1)$$

where X_t represents time periods (X_t = 1, 2, ... n), S_j are the quarterly seasonal indices repeated each year, and C_t are cyclical indices. These time series components are described in most elementary statistics textbooks. In general, least squares predictions by (1) are more accurate than non-least squares estimates obtained by the time series decomposition method;

$$T_tS_jC_t, = T_t{*}S_j{*}C_t, \qquad (2)$$

where S_j and C_t are defined in (1) above. T_t are trend estimates; $T_t = b_0 + b_1X_t$. When the accuracy of (2) approximates that of (1), combine the multiplicative forecast of (2) with the additive forecast of (1);

$$\hat{Y}_t = b_0 + b_1X_t + b_2S_j + b_3C_t + b_4\, T_tS_jC_t. \qquad (3)$$

In (3), the additive (1) and multiplicative (2) forecasts are combined for improved accuracy.

Indicator Variables. Approximately the same accuracy of (1) is obtained using indicator variables;

$$\hat{Y}_t = b_0 + b_1X_t + b_2D_2 + b_3D_3 + b_4D_4 + b_5C_t, \qquad (4)$$

where D_j = 1 if quarter j, j = 2, 3, 4, 0 otherwise, and X_t and C_t are defined in (1). The indicator variable method of including seasonal variation is described in most statistical, forecasting, and econometric textbooks. However, their use is not without severe limitations. Eleven indicator variables must be employed to represent monthly seasonal variation; 22 are needed if interaction is involved. Hence, indicator variables for explaining seasonal variation soon become impractical making seasonal indices the preferred explanatory variable for use in forecasting models.

Benefits. This paper reveals the benefits and accuracy of employing a combined forecast model with a single seasonal index variable rather than multiple indicator variables. It also explains how greater accuracy is obtained by employing unrestricted least squares. Since influences of the past may not necessarily continue in the future, the authors promote the use of structured judgmental modifications for increasing the accuracy of out-of-sample predictions. These improvements not only produce practical forecasting models with increased accuracy for future predictions but also enjoy the pedagogy of spreadsheet regression.

COMPARING LEAST SQUARES SEASONALS WITH INDICATOR VARIABLES

This section reveals how least squares seasonals are more practical and parsimonious than indicator variables. Least squares seasonal are traditional seasonal indices used as an explanatory variable in a least squares forecasting model. The inclusion of X_t, S_j and C_t in (1) is easily justified. When combining forecasts, time series components (trend, seasonal, and cyclical) along with other forecasts generally make a significant contribution in explaining the behavior of the response variable Y_t.

Least Squares Seasonals. Although the same approximate accuracy is obtained from (1) and (4), there is a difference. For example, the indicator variable model

$$\hat{Y}_t = b_0 + b_1 D_1 + b_2 D_2 + b_3 D_3 \tag{5a}$$

uses quarterly means $\bar{Y}_{\bullet j}$ in describing seasonal variation;

$$\hat{Y}_t = \bar{Y}_{\bullet 4} + (\bar{Y}_{\bullet 1} - \bar{Y}_{\bullet 4})D_1 + (\bar{Y}_{\bullet 2} - \bar{Y}_{\bullet 4})D_2 + (\bar{Y}_{\bullet 3} - \bar{Y}_{\bullet 4})D_3 \tag{5b}$$

where $D_j = 1$ if quarter j, j = 1, 2, 3, 0 otherwise. Observe, when all $D_j = 0$, $\hat{Y}_t = \bar{Y}_{\bullet 4}$; also, in representing quarter 1, D_1=1, D_2=D_3=0, and (5b) becomes $\hat{Y}_1 = \bar{Y}_{\bullet 1} = \bar{Y}_{\bullet 4} + (\bar{Y}_{\bullet 1} - \bar{Y}_{\bullet 4})$.

Approximately the same accuracy of (5) is obtained from $\hat{Y}_t = b_0 + b_1 S_j$ that can be represented as

$$\hat{Y}_t = [\bar{Y}_{\bullet\bullet} - b_1 \bar{S}] + b_1 S_j \quad = \bar{Y}_{\bullet\bullet} + (S_j - 1)b_1 \tag{6}$$

where S_j are quarterly indices, the mean $\bar{S}$= 1, and $\bar{Y}_{\bullet\bullet}$ is the overall mean. Observe, if S_j is 1.15, the predicted quarterly value is represented as $\bar{Y}_{\bullet\bullet} + 0.15 * b_1$ -- the overall mean plus 15% of the slope. The slope b_1 is the change in Y_t given a unit change in S_j. In most time series, $\bar{Y}_{\bullet j}$ and $\bar{Y}_{\bullet\bullet} + (S_j - 1)b_1$ are not appreciatively different. Hence, the same approximate accuracy is usually obtained from (5) and (6). The above symbolic equations become more complex when other variables are included in the model.

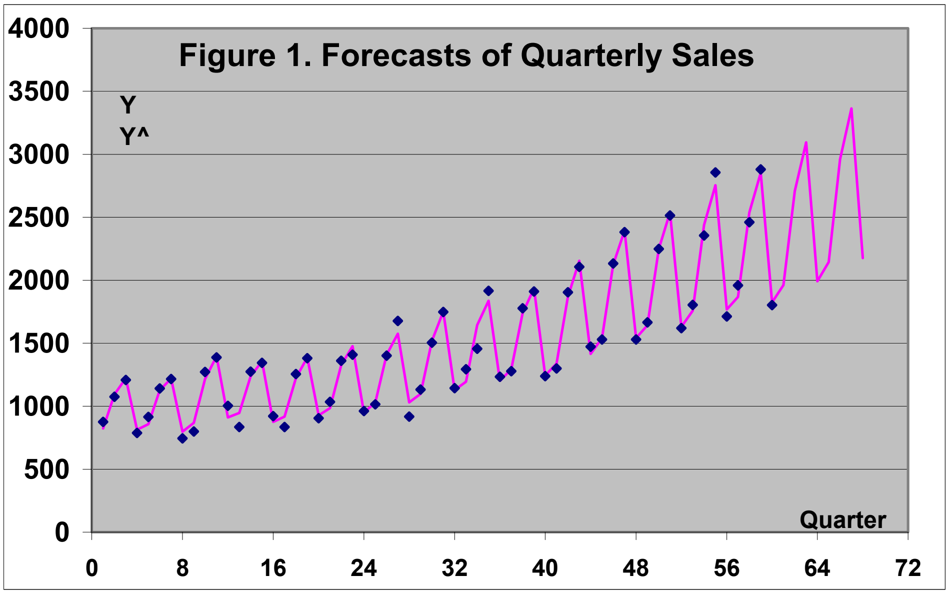
Interaction. When seasonal variations grow larger with time (see Figure 1), include interaction between the trend X_t and the seasonal indices S_j:

$$\hat{Y}_t = b_0 + b_1 X_t + b_2 S_j + b_3 C_t + b_4 X_t S_j. \tag{7a}$$

If X_t and C_t are held constant, a change in Y_t given a change in S_j is dependent upon the value of X_t:

$$\hat{Y}_t = [b_0 + b_1 X_t + b_3 C_t] + [b_2 + b_4 X_t] S_j. \tag{7b}$$

A forecasting equation with trend, seasonal, cyclical, and interaction requires 24 variables if indicator variables rather than a single seasonal index variable is used with monthly data. Observe, the interaction term ($X_t * S_j$) in (7) often approximates the decomposition of a multiplicative time series forecast described by (2). In Figure 1 below, seasonal variation diverges and an interaction term $X_t S_j$ is employed with significance.



Moving Seasonals. When using a constant seasonal index, it is assumed that seasonal variation is not moving – does not possess a trend. However, if this assumption is incorrect, a moving seasonal index needs to be constructed. These indices are described in older textbooks and are still used by the U.S. Bureau of the Census. They are used when average seasonal indices do not adequately describe current seasonal variations. When forecasting future values of Y_t, moving seasonal indices may be obtained subjectively. This allows the model to possess structured judgment modification capabilities. Young (1982), among others, believe that judgmental modification is an essential part of forecasting. Judgmental modification is the forecaster injecting personal knowledge of the subject matter into the forecast (Edmundson, 1990). This can be accomplished in numerous ways, one of which is by forecasting future values with moving seasonal indices and cyclical indices obtained subjectively. A description concerning the use of subjective cyclical indices used in the forecasts given below can be found at www.wtamu.edu/~flandram/SUP_DSJIE.pdf. Moving seasonal indices are described in older textbooks such as Croxton and Cowden (1955).

COMBINING FORECASTS

Granger and Ramanathan (1984) argue that combined forecasts from several methods outperform forecasts from a single method. They point out that values from discarded forecasting models still contain useful information about the underlying behavior of Y_t. When biased forecasts are included in a least squares equation, the intercept adjusts for the bias. Hence, in the combination process, it is important to include the intercept—unrestricted least squares—and let least squares automatically assign weights to the forecasts.

Example

The example illustrated by Albritton and McMullen (2006) concern quarterly sales data with a curvilinear trend, substantial seasonal variation that diverges ("fans out"), and a slight cyclical movement (see Figure 1). A forecast from Excel Solver produces a sum of squares error of $SSE = \Sigma(Y-\hat{Y})^2 = 266{,}592$. These authors reveal this forecast is 13% smaller than the time series decomposition forecast of SSE = 306,469. However, (8) in Table 1 reveals this time series component model (with interaction $X_t{*}S_j$) reduces SSE to 210,732. When the multiplicative forecast $T_t{*}S_j{*}C_t$ is included (models (9) and (10) in Table 1), SSE is further reduced to 186,253 and 187,765, respectively. Since the R^2 value for all equations in Table 1 possess an approximate value of $R^2= 0.98$, the configuration in Figure 1 is the same for all equations. The root mean square error, $RMSE = (SSE/n)^{1/2}$, shows a range from 71.5 to 55.7. The multiplicative forecast $T_t{*}S_j{*}C_t$ captures both the curvilinear trend and cyclical movement. Therefore, the X^2 and C_t additive terms are deleted; (9) reduces to (10). Since parsimony has merit, (10) is the forecasting model of choice.

Table 1. Comparing Combined Forecasts

MODEL	no.	SSE	RMSE	R^2
Time Series Decomposition		306,469	71.5	0.981
SOLVER		266,592	66.7	0.983
$\hat{Y}_t = b_0 + b_1X_t + b_2 X_t^2 + b_3S_j + b_4C_t + b_5X_tS_j$	(8)	210,732	59.3	0.987
$\hat{Y}_t = b_0 + b_1X_t + b_2 X_t^2 + b_3S_j + b_4C_t + b_5T_tS_jC_t$	(9)	186,253	55.7	0.989
$\hat{Y}_t = b_0 + b_1X_t + b_2 S_j + b_3T_tS_jC_t$	(10)	187,765	55.9	0.988

where $T_t = b_0 + b_1X_t + b_2 X_t^2$

DISCUSSION AND CONCLUDING REMARKS.

When forecasting, use caution in allowing low in-sample SSE values to be the sole criterion. Emphasis is needed on accurate out-of-sample prediction measures. Remember, quality of fit does not guarantee quality of predictions. Also, the SSE value associated with Excel Solver will be even lower by including the cyclical time series component. Finally, influences of the past will not usually continue to exert the same degree of influence in the future. Therefore, most forecasts benefit from structured human judgmental modification such as the use of moving seasonal indices (Edmundson, 1990). These types of subjective indices increase the accuracy of future predictions and are easily employed in unrestricted least squares equations.

Teaching. Most operations management textbooks include time series decomposition and regression in their forecasting chapter. After calculating seasonal indices, it is extremely easy to use these indices as an explanatory variable in a least squares forecasting model. Once students realize that time series components can be used in least squares forecasting, statistical modeling is taught. This includes the use of (a) an interaction variable when the seasonals diverge (see Figure 1); (b) higher order trend variables when the relationship is curvilinear; and (c) event modeling techniques. When covering time series analysis in an elementary statistics course, teach students that time series components (trend, seasonal and cyclical components) can easily be combined in a least squares forecasting equation such as (1). In the elementary statistics course, continuity is then continued between the regression and time series chapters.

At one author's university, students come into the required junior level operations management course with a knowledge of Excel, regression and time series. Nevertheless, a review of Excel, regression, and scatter diagrams is conducted the first week—two class periods. Three chapters later, the forecasting chapter is covered. When covering this chapter, a review of time series components and their application in least squares forecasting models only takes two class periods. Again, they are taught to calculate seasonal and cyclical indices and then include these indices as explanatory variables in a least squares equation. An application phase of forecasting with empirical data—such as Walmart, The GAP, domestic car sales, housing starts—takes another two class periods. Some statistical modeling techniques are used when forecasting with empirical data. Hence approximately three weeks are spent covering forecasting. This includes the first week review and the two weeks spent on the forecasting chapter. Feedback from students reveals that the empirical applications phase builds student interest, confidence, and a sense of accomplishment early into the course. Teaching forecasting applications in the required operations management course also motivate students in elementary statistics. Students are aware that statistical methods taught in elementary statistics are later used in other required courses.

Conclusion. The practical and educational innovations of this article are summarized below:

(a) Students find an easy transition of applying the time series decomposition concept learned in elementary statistics to combining forecasts with time series components. This transition makes the forecasting phase of a management science or operations management course more interesting, practical, and enjoyable.

(b) Combining forecasts with time series components enable students to become more adapt in the statistical modeling of higher order, interaction, and judgmental modification variables.

(c) In advance courses, students realize that exponential smoothing and Box-Jenkins ARIMA forecasts can also be combined with time series components to produce practical and superior forecasting models.

(d) The forecasting models described above are derived while enjoying the pedagogy of spreadsheet regression rather than employing "black box" esoteric statistical forecasting software.

(e) The innovative use of time series seasonal components S_j rather than the traditional indicator variables D_j greatly expands the modeling capabilities of unrestricted least squares forecasting equations.

These educational and practical innovations will enable combined forecasts with time series components to become increasingly popular in the classroom and in the workplace. Again, using seasonal and cyclical indices in a least squares model is destined to become a routine operation in forecasting.

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