Pressure, Radiation

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Radiation pressure

Electromagnetic waves (or photons) carry momentum.

Since the irradiance is energy/area/time, pressure is the energy density of a wave. We can express this with the average magnitude of the Poynting vector $S$.

$$\langle P(t) \rangle_T = \frac{\langle S(t) \rangle_T}{c} = \frac{I}{c}$$

in N/m$^2$. This is for a perfect absorber. For a perfect reflector it is $2I/c$ (remember elastic collisions?)
Each photon carries an energy $\mathcal{E} = h\nu$. The photon will carry momentum

$$p = \frac{\mathcal{E}}{c} = \frac{h}{\lambda}.$$ 

In vector form

$$\vec{p} = \hbar \vec{k}$$

$\vec{k}$ is the propagation vector and $\hbar \equiv h/2\pi$. 
Effects of radiation pressure. Average flux of energy from the Sun at Earth’s orbit is 1400 W/m$^2$. This gives a pressure of about 9 N/km$^2$.

This is sufficient pressure to affect the orbit of long-range spacecraft, and must be accounted for in mission planning. It is also possible to levitate very small objects using radiation in the laboratory (see fig. on p. 57). A sail of about a square km or more could be used to accelerate a few tons across the inner solar system.

In stellar interiors, the radiation pressure is a major part of the structure of the star (such as our Sun).
Radiation—if a charge moves nonuniformly, it radiates.

(See figs. 3.27–28.)

The radiation fields (large $r$) due to acceleration go as

\[ \vec{E} \sim \vec{r} \times (\vec{r} \times \vec{a}) \] and
\[ \vec{B} \sim \vec{a} \times \vec{r}, \]

where $\vec{r}$ points from the charge and $\vec{a}$ is the acceleration vector.

This means that energy is most strongly radiated perpendicular to the acceleration causing it.
Synchrotron radiation

Any curved path means an acceleration.

A free charge on a curved path will radiate.

This is important astronomically due to charges in stellar and interstellar magnetic fields, and a charged particle accelerator driven as a synchrotron source can produce intense X-rays. See fig. 3.30.
Electric dipole radiation

(See fig. 3.32.)

A dipole of charge with static dipole moment $\hat{p}_0 = qd$ oscillating at angular frequency $\omega$ produces a radially outward irradiance

$$I(\theta) = \frac{\hat{p}_0 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2 \theta}{r^2},$$

for $r \gg d$, the radiation zone.

Note that

$$I \sim \omega^4 \sim \nu^4 \sim \frac{1}{\lambda^4}.$$  

Scattering of radiation by the dipole moment of air molecules works similarly—thus the sky scatters blue light ($\nu$ for blue is higher than red).
**Atomic emission and absorption**

Since the energy levels of atomic electrons are quantized, the amount of energy an atom can absorb when an electron jumps to a higher state is quantized.

An single atom can only absorb a photon of $\mathcal{E} = h\nu$ where $\mathcal{E}$ matches the energy difference between some pair of accessible states.

Likewise the energy of a photon emitted must match the energy between two states of the atom when it makes a transition to a lower energy level:

$$\Delta\mathcal{E} = h\nu.$$