A maritime analogy of the Casimir effect

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At sea, on a windless day, in a strong swell, free floating ships will roll heavily. It was believed in the days of the clipper ships that under those circumstances two vessels at close distance will attract each other. Do they? The ships are harmonic oscillators in a wave field and as such analogous to two atoms in the sea of vacuum fluctuations. These atoms do attract: the van der Waals force.

I. INTRODUCTION

Early in the 19th century, P. C. Causseé published in France his book "Album of the Mariner" containing drawings of situations at sea and directions on how to handle in these situations. Figure 1 shows Causseé's "Calme avec grosse houle," no wind any more but still a big swell running.

Eventually the ships end up lying parallel to the wave crests and start rolling heavily. Causseé warns of the situation of two ships coming at close distance. Then "une certaine force attractive" will pull the ships toward each other. The rigs of the rolling ships become entangled and we have a disaster.

Causseé recommends the lowering of a sloop with a sufficiently strong crew to tow one ship out of reach of the other.

Hazelhoff Roelfzema of the Amsterdam Shipping Museum, who drew my attention to Causseé's book,[1] wonders what kind of force this might be. Does it really exist or is it only the fantasy of sailors? In the threatening situation of Fig. 1 sailors could easily think they are attracted to the danger.

II. SHIP AS AN HARMONIC OSCILLATOR

A. Attractive force or superstition?

If the force is real, we must be able to calculate it. This could be done by brute force by the interaction of the wave field and ships, but that would be too much work and not very satisfying either, for no insight is gained. It is far more rewarding to look for some mechanism.

Two ships on a swell are two harmonic oscillators, dipole radiators, in a wave field. This brings to mind an article by Daniel Kleppner[2] in *Physics Today* of October 1990. Kleppner refers here to an article of Casimir[3] that deals with an attractive force between two polarizable atoms. Kleppner[2] considers a system of two weakly coupled LC resonant circuits at distance R. Due to the coupling the single resonant frequency splits, lifting the degeneracy. The argument proceeds quantum mechanically, the frequency splitting affects the zero point energy $\frac{1}{2}h\nu$ of both oscillators, one up, one down. But the total energy changes somewhat, a higher-order term leads to an attractive force, the van der Waals force. Casimir[3] derived the attractive force between polarizable atoms in 1948 and found a simple expression. This led him to a more general way of reasoning. In a subsequent paper Casimir[4] replaced the two atoms by two perfectly conducting parallel plates at close distance. Now the mode structure of the vacuum fluctuations in the narrow space between the plates differs from that outside, due to the conducting boundary conditions. The boundary conditions decrease the mode density of the vacuum energy in the inner space. This leads to an attractive force between the plates. The referee mentioned a highly theoretical, very elaborate, article by Plunien *et al.*[5] on the Casimir effect and related subjects. Plunien gives 156 references.

II. RADIATION PRESSURE

This quantum mechanical reasoning is of course unsuitable as an analogy for our ships. Figure 1 really does not invite a quantum treatment. But Casimir's plate attraction can also be formulated in terms of radiation pressure. The energy density, and therefore the radiation pressure, is less between the plates than on the outside, yielding a net attraction. Zero point energy is not a suitable quantity for ships but radiation pressure certainly is. Radiation pressure is not restricted to quantum mechanics, it is also a classical quantity. Radiation pressure is a universal phenomenon, it holds for all kinds of waves: the electromagnetic waves between Casimir's plates, sound waves[6] and also sea waves.

If a current element $\mathbf{i}=q\mathbf{v}$ is placed in a traveling electromagnetic wave with an $\mathbf{E}$ and a $\mathbf{B}$ it will tap a power from the wave $W=\mathbf{i}\cdot\mathbf{E}$. At the same time it experiences a Lorentz force $\mathbf{K}=\mathbf{i}\times\mathbf{B}$. For the magnitudes of $\mathbf{E}$ and $\mathbf{B}$ we have $E=cB$, so the radiation force K in the direction of propagation

$$K=W/c. \tag{1}$$

As this is already true for the instantaneous force it will certainly hold for the average force over a number of periods, the radiation pressure proper. Expression (1) is true for both electromagnetic waves and transverse mechanical waves like sea waves. In (1) c is the wave velocity of the particular wave type at hand.

In Fig. 2 the sea surface is modeled by a string and the ship by a platform ("catamaran"). The ship is coupled to the "sea" by two frictionless ringlets around the string. Now the string can exert a torque D on the ship.

A transverse wave moves to the right:

$$y(x,t)=y(x-ct). \tag{2}$$

The power exchange between string and ship is not governed by y itself but only by the slope $\phi=\partial y/\partial x$. ϕ is also a traveling wave:

$$\phi(x,t)=\phi(x-ct). \tag{3}$$

Due to the string's curvature, the torque D produces a horizontal force, the "radiation pressure:"

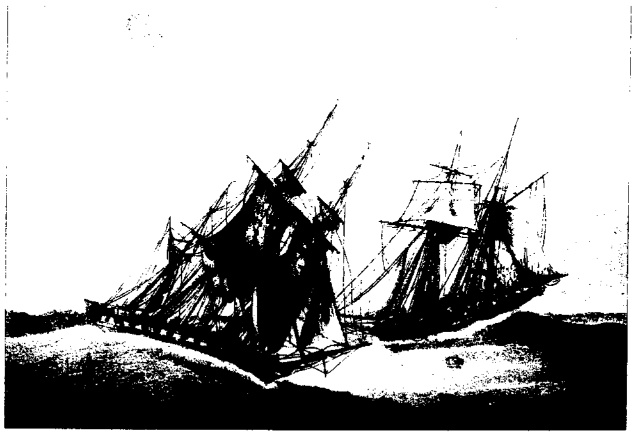

Fig. 1. Two ships roll heavily on a long swell and there is no more wind to damp their rolling. In this situation a strange force, "une certaine force attractive," will pull the two ships toward each other. From P. C. Causseé: "the Mariners Album," early 19th century.

$$K=D\cdot\frac{\partial\phi}{\partial x}. \tag{4}$$

The torque D delivers a power W to the ship (D times angular velocity)

$$W=D\cdot\frac{\partial\phi}{\partial t}=D\cdot\frac{\partial\phi}{\partial x}\cdot\frac{\partial x}{\partial t}. \tag{5}$$

The constancy of the wave shape (3) gives $\partial x/\partial t=c$ and combined with (4) and (5) yields a radiation pressure in the direction of propagation

$$K=W/c. \tag{6}$$

This radiation pressure expression is identical with (1) for the electromagnetic wave in Casimir's cavity. In this derivation of radiation pressure no use is made of the wave mechanism or even the wave equation. Only the constancy (3) of the wave *shape* is used. Therefore (6) also holds for a sea wave, a wave that has a different wave mechanism.

But sea waves are dispersive; an arbitrary wave shape y does not remain constant. Fortunately a rolling ship emits sine waves and that shape does remain constant. So the c in (6) is the *phase* velocity belonging to the period T, that is $c=gT/2\pi$ with g the acceleration of gravity.

A ship that rolls on a swell absorbs power from the wave. It experiences a radiation force that causes it to drift to leeward. But as both ships in Fig. 1 will drift nearly the same amount this effect is of little consequence for their relative positions.

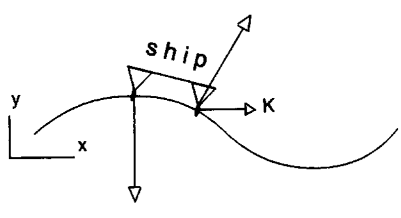

Fig. 2. A transverse wave in a string models the sea surface. The "sea" exerts a torque D on the "ship" by means of two frictionless ringlets around the string.

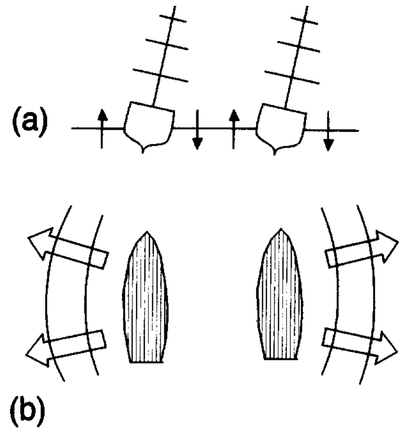

Fig. 3. (a) Two ships at close quarters roll on a long swell. (b) They re-emit the absorbed power as secondary waves.

IV. TWO SHIPS AS CASIMIR PLATES

A rolling ship re-emits the absorbed power as secondary waves. A single ship emits its waves over 360° and the reaction forces on the ship cancel. For two ships the situation is different, Fig. 3(a). If the primary wavelength is long compared with the distance between the ships, they will roll in phase. The port wave will be in phase with the other port wave and starboard is in phase with starboard. But the sides facing each other are in anti-phase and hence the waves that the ships radiate toward each other cancel. Due to this interference there is little or no radiated wave energy in the space between the ships, just as there is less energy in the space between the Casimir plates. Now the radiation reaction forces of the outwardly emitted waves [arrows in Fig. 3(b)] are no longer canceled by the inside radiation pressure, and therefore the ships are pushed toward each other, exactly as the Casimir plates are. So there really is an attractive Causseé force and it is roughly given by the wave power that is reradiated by the ship, divided by c.

This power is easily found from the observed roll amplitude and the ship's design parameters. A rolling ship is a wave making machine of high efficiency; hence the radiated power is found as the loss, the dissipation, of the oscillator energy. The energy content of the ship as an oscillator is

$$E=\tfrac{1}{2}SA^2 \tag{7}$$

with $A=$ roll angle amplitude and $S=mgh=$ ship's stiffness, that is, the restoring torque per radian roll angle. Here m is the ship's mass, g the acceleration of gravity, and h is a ship parameter, the metacentric height. h is the length of a pendulum that has the same stiffness S as the ship and a bob with the same mass m as the ship.

If Q is the quality factor of the oscillator and T the period, the energy loss, the radiated power is

$$W=2\pi E/(QT). \tag{8}$$

This yields for the attractive force

$$F = 2\pi^2 \eta m h A^2/(QT^2), \quad (9)$$

where the efficiency factor η takes account of friction losses and the divergence of the reradiated wave.

F was calculated for two clipper ships of 700 tons of metacentric height $h = 1.5$ m, rolling amplitude $A = 0.14$ Rad (8°) with period $T = 8$ s, $Q = 2.5$, and $\eta = 0.8$. They attract each other with a force of 2000 N. This is a perfectly acceptable result. With 200 N there would be hardly any effect, and with 20 000 N, Causseé's remedy of towing by a sloop would have been of no avail. But 2000 N can still be overcome by rowers, especially when towing at an angle.

Causseé's condition of perfect calm also makes sense: already in a light breeze the sails will damp the oscillations effectively, and as F scales with A^2 the force F diminishes rapidly.

The old tales were true. Rolling ships do attract each other. Two ships on a wavy sea attract each other as two atoms do in the sea of vacuum fluctuations.

[1]P. C. Causseé, *l'Album du Marin* (Nantes, Charpentier, 1836).

[2]D. Kleppner, "With Apologies to Casimir," Phys. Today **43**(10), 9–11 (1990).

[3]H. B. G. Casimir and D. Polder, "The Influence of Retardation on the London-van der Waals Forces," Phys. Rev. **73**(4), 360–372 (1948).

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Rolling and slipping down Galileo's inclined plane: Rhythms of the spheres

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In "Two New Sciences" (TNS) Galileo presents a number of theorems and propositions for smooth solid spheres released from rest and rolling a distance d in time t down an incline of height H and length L. We collect and summarize his results in a single grand proportionality P: $d_1/d_2 = (t_1^2/t_2^2)(H/L)_1/(H/L)_2$. ($P$) From what he writes in TNS it is clear that what we call P is assumed by Galileo to hold for all inclinations including vertical free fall with $H/L = 1$. But in TNS he describes only experiments with *gentle* inclinations $H/L < 1/2$. Indeed he cannot have performed the vertical free fall ($H = L$) experiment, because we (moderns) know that as we increase H/L, P starts to break down when H/L exceeds about 0.5, because the sphere, which rolls without slipping for small H/L, starts to slip, whence d starts to exceed the predictions of P, becoming too large by a factor of 7/5 for vertical free fall at $H/L = 1$. In 1973 Drake and in 1975 Drake and MacLachlan published their analysis of a previously unpublished experiment that Galileo performed that (without his realizing it) directly compared rolling without slipping to free fall. In the experiment, a sphere that has gained speed v_1 while rolling down a gentle incline is deflected so as to be launched horizontally with speed v_1 into a free fall orbit discovered by Galileo to be a parabola. The measured horizontal distance X_2 traveled in this parabolic orbit (for a given vertical distance fallen to the floor) was smaller than he expected, by a factor 0.84. But that is exactly what we (moderns) expect, since we know that Galileo did not appreciate the difference between rolling without slipping, and slipping on a frictionless surface. We therefore expect him to predict X_2 too large by a factor $(7/5)^{1/2} = 1/0.84$. He must have been puzzled. Easy "home experiments" with simple apparatus available to Galileo (no frictionless air tracks, strobe lamps, or electronic timers!) allow the student to use his/her musical ear (for rhythm and tempo) to study vertical free fall as well as balls rolling down steep or gentle inclines, with or without slipping, and perhaps appreciate Galileo's dilemma. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

In the middle of a wonderful essay entitled "The Aesthetic Equation,"[1] Professor Hans Christian von Baeyer writes that Galileo showed that

The ratio of the acceleration of a ball rolling down an incline to the acceleration of a ball in a free fall is equal to the ratio of the height of the incline to its length. (a)

What a remarkable and unexpected equality between two dimensionless ratios! It reminds one of Archimedes' beauti-